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Fuzzy Decision-Making in Health Systems: A Resource Allocation Model

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Abstract The efficient use of resources in health systems is important due to the increasing demand and limited funding. Large health systems often have fixed input resources (such as budget and staffing) to be allocated among individual hospitals/clinics with particular target output levels. We propose an optimization model with fuzzy constraints that can be used for automatic resource re-allocation with respect to different levels of risk preferences. We illustrate its applicability using data from a U.S. Army hospital network. The implications of the proposed fuzzy decision-making model for healthcare decision-makers and its relevance to health-care policy and management are also discussed.

Keywords Multi-objective optimization · fuzzy modeling · resource allocation · health systems · military medicine

1 Introduction

The burden on public, private and military health systems has increased due to both population growth and limited funding. Large health systems are challenged to provide health services at a certain quality level with a fixed amount of resources. For instance, the Military Health System (MHS) of the U.S. Department of Defense has become a $52 billion network that provides health services to over 4.5 million enrolled uniformed service members, their family members, survivors, and retirees [1]. Due to the rising health care demands and costs, the MHS needs to effectively allocate (and re-allocate) resources by balancing costs, providers, clinical visits and inpatient/outpatient workload across the MHS. Therefore, health care decision-makers are in search of analytical methods that systematically deal with these strategic and policy challenges and utilize the existing resources while conforming with target performance levels. This paper proposes an optimization model with fuzzy constraints that can help decision-makers perform sensitivity analysis and automatically re-allocate system input resources for different levels of risk preferences.

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1.1 Literature Review

Charnes et al. [2] were among the first to apply advanced performance measurement methods in U.S. Army health care facilities. They used data envelopment analysis (*DEA*) [3] to analyze the relationship among outputs (number of trained personnel, relative weighted product (RWP, a weighted inpatient workload metric), and clinic visits) and inputs (full time equivalent (FTE) employees, inpatient expenditures, outpatient expenditures, weighted procedures, occupied bed days, and operating room hours). Ozcan et al. [4] performed a longitudinal study of 124 MHS hospitals to evaluate trends in hospital efficiency using data from the American Hospital Association Survey. Piner [5] used *DEA* to compare efficiency of the clinics with respect to the staffing and expenses, and found larger hospitals to be more efficient. Simultaneous measurement is also an important component for the performance evaluation of public health care organizations. The balanced scorecard methodology presented by Grigoroudis [6] was one such approach. They considered financial performance indicators in addition to the non-financial performance indicators such as the service quality, customer satisfaction, competition power, social character, and self-improvement ability of the organization. Note that all these studies provide performance measurement, as well as sensitivity analysis but none of them provide direct decision support.

Decision-making approaches utilized for performance-based resource allocation in the healthcare sector have a great range from optimization to the analytic hierarchy process. Eichler et al. [7] provided an overview of the cost-effectiveness analysis for healthcare resource allocation decision-making. Kwak et al. [8] proposed a goal programming model that allows decision-makers to make strategic planning and allocation decisions with limited human resources in a healthcare system. Specifically, their model assigns the personnel to the shift hours with the objective of minimizing total payroll costs subject to the patient satisfaction constraints. Kwak et al. [9] presented an application of multi-criteria mathematical programming to allow for strategic planning for business process infrastructure development in the healthcare system. They used the analytic hierarchy process to identify and prioritize the goal levels. Aktas et al. [10] proposed a management-oriented decision-support model to assist health system managers in improving the efficiency. They identified key variables of system efficiency and employed a Bayesian belief network to model the causal relationships.

Fuzzy decision-making models are also used for performance-based resource allocation. Hussein et al. [11] introduced a fuzzy dynamic programming model for multiple criteria resource allocation problems, whereas Mjelde [12] considered the problem of allocation of fuzzy resources to fuzzy activities. In terms of applications to the healthcare sector, the integer programming method of Kachukhasvili et al. [13] is an example that minimizes the total waiting time of patients while using fuzzy sets to group resources. Arenas [14] evaluated hospital service performance by using a fuzzy linear goal programming model with crisp parameters.

These models do not utilize *DEA* within a decision-making framework. *DEA* provides a straightforward method to analyze and quantify the sources of inefficiency for multiple inputs and outputs. The use of *DEA* as part of direct decision support methods for the MHS dates back to the study of Fulton et al. [15]. They used *DEA* and stochastic frontier analysis to identify the cost drivers for performance-based resource allocation. Then, Fulton et al. [16] proposed regression based military hospital cost models that included *DEA* efficiency scores in addition to the variables of quality, access, and efficiency.

1.2 Motivation and Overview

These aforementioned approaches handle resource allocation problems from different perspectives and provide sensitivity analyses. However, the evaluation of slack and reduced costs in traditional multiple criteria mathematical programming and *DEA* models do not provide sufficient decision support for re-allocation of system resources. For centrally funded organizations such as the MHS, it is important to efficiently re-allocate system resources since total funding is fixed and they are under pressure to sustain health system output objectives.

Bastian et al. [17] proposed a hybrid *DEA*-based optimization model that helps the decision-maker balance competing objectives automatically. They used the structural similarities in the two multi-criteria decision-making methods [18, 19] to provide multiple criteria decision-support within a military hospital system. This so-called auto-optimization model provides MHS decision-makers with a decision support tool for re-allocation of input resources within a fixed-resource setting. Specifically, their multi-objective optimization model adjusts resources automatically across all treatment facilities to achieve maximum system efficiency while achieving a minimum level of performance for each hospital. Their assumption is that the minimum level of performance (i.e., global technical efficiency variable) is fixed and equal for all hospitals. One limitation of their approach is the assumption of deterministic health system parameters and resources, which may not be realistic within a *DEA* setting since possibilistic uncertainty of the physical units was not considered. Further, it does not allow the decision-makers to take into account different risk levels.

A general drawback with the traditional *DEA*-based analysis is the assumption that system inputs and outputs are fundamentally crisp. *DEA* is very sensitive to possible data errors since it focuses on frontiers or boundaries [20]. However, some or all of these variables of interest in real-world settings include some degree of imprecision or ambiguity. The source of this imprecision might be related to incomplete, non-obtainable or non-quantifiable information. Hence, methods that allow the decision-maker to deal with imprecise data become crucial, especially in sensitive decision-making environments [21]. This imprecision can be represented with well-defined bounded intervals, ordinal (rank order) data or fuzzy numbers [22, 23]. *Fuzzy set theory* defines the inherent uncertainty that is mostly encountered in the physical systems as possibilistic (or linguistic) uncertainty rather than probabilistic uncertainty. Essentially, a fuzzy framework provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables [24]. Thus, intuitively plausible semantic description of the imprecise properties of data used in the natural systems might be accomplished by fuzzy theory [25]. This framework also allows the decision-makers to express or measure the imprecision of data using particular fuzzy membership functions. These membership functions reflect the satisfaction and risk level of decision-makers and can be defined with respect to strategies.

Sengupta [26,27] was the first to incorporate fuzziness into a *DEA* model by defining tolerance levels on both the objective function and constraint violations. This work was followed by a number of fuzzy data envelopment analysis (*FDEA*) studies in which imprecision or vagueness included by input and output data are handled in the context of fuzzy linear or non-linear programming. A comprehensive and recent overview of the *FDEA* models has been provided by Hatami-Marbini et al. [28]. *FDEA* approaches can be categorized into six groups: the tolerance approach, the α-level based approach, the fuzzy ranking approach, the possibility approach, the fuzzy arithmetic, and the fuzzy random/type-2 fuzzy set [29]. With regards to the *FDEA* based resource allocation models, the fuzzy goal *DEA* framework of Sheth and Triantis [30] is such an application. Further, Uemura [31] used a fuzzy goal based on the evaluation ratings of individual outputs obtained from the fuzzy log-linear analysis in the context of *DEA*. Zerafat Angiz et al. [32] introduced an alternative ranking approach based on *FDEA* to aggregate the preference rankings of decision-maker groups.

This paper aims to overcome the aferomentioned shortcomings by using a fuzzy decision-making approach based on a *DEA*-based auto-optimization model with fuzzy constraints. Particularly, we introduce a fuzzy global technical efficiency variable into the model of Bastian et al. [17] that results in a hybrid auto-optimization model with fuzzy constraints. This model corresponds to a fuzzy mathematical programming model with fuzzy constraints, and it is transformed into a crisp model by means of fuzzy operators as described in Zimmermann [33] and Werners [34]. The proposed approach indeed utilizes the max-min operator defined by Bellman and Zadeh [35] to obtain the optimal decision in context of the fuzzy programming. While doing so, the global technical efficiency variable is assumed to be a fuzzy number, and its fuzzy membership function is constructed using the approach of Wang and Fu [36]. This allows the decision-maker to model the ambiguity of the global technical efficiency variable. In addition, this framework allows us to consider various risk profiles of decision-makers, particularly risk seeking, risk neutral and risk averse strategies (for a similar application with various risk profiles, see the portfolio management model of Keskin et al. [37]).

This paper is structured as follows. In Section 2, we introduce the model and Section 3 provides a computational experiment comparing traditional *DEA* with the new efficiency measurement and decision-making methods. Section 4 concludes the paper with a discussion relevant to health care policy and management.

2 Materials and Methods

This section first reviews the basic concepts of *DEA* and then revisits the multi-objective auto optimization model (*MAOM*) of Bastian et al. [17], and extends it to introduce the proposed fuzzy multi-objective auto optimization model (*FMAOM*). *MAOM* is a resource allocation-based optimization model where decision-makers can perform sensitivity analysis and re-allocate system input resources automatically within a fixed-input MHS. Whereas *FMAOM* introduces a fuzzy global technical efficiency variable into this model and is able to handle different risk strategies. Detailed descriptions of all models are in the following subsections.

2.1 Data Envelopment Analysis (*DEA*)

DEA is a set of flexible, mathematical programming approaches for the assessment of efficiency, where efficiency is often defined as a linear combination of the weighted outputs divided by a linear combination of the weighted inputs as in the Charnes, Cooper, and Rhodes (CCR) model [3], which is a constant returns-to-scale formulation. Assume that an organization wishes to assess the relative efficiencies of some set of comparable sub-units, so called Decision Making Units (DMUs). For each DMU, there is a vector of associated inputs and outputs of managerial interest [38].

The decision-maker is interested in either maximizing the outputs while not exceeding current levels of inputs (output oriented) or minimizing the inputs without reducing any of the outputs (input oriented). The decision-maker assumes that the traditional definition of engineering efficiency (ratio of weighted outputs to weighted inputs) will result in an acceptable solution for technical efficiency. With these assumptions in place, one may formulate the following fractional programming problem that may be solved to determine technical efficiency, defined (for now) as the ratio of weighted outputs to weighted inputs, for each separate DMU. The following is known as the input-oriented CCR constant returns to scale *DEA* model:

$$\max \quad \theta = \frac{u^T y_o}{v^T x_o} \tag{1}$$

$$\text{subject to} \quad \frac{u^T y_z}{v^T x_z} \leq 1 \quad \forall z \tag{2}$$

$$u \geq 0 \tag{3}$$

$$v \geq 0 \tag{4}$$

In this formulation, there is a vector of outputs (y), a vector of inputs (x), and z DMUs. Efficiency is designated as θ. The index o identifies the selected DMU for which an efficiency score will be generated. This mathematical program is run z times (the total number of DMUs), once to determine the efficiency of each DMU. While multiple objective linear programming simultaneously solves multiple objective functions given a value function, *DEA* optimizes efficiency for an individual DMU. The components of the vectors are the weights to be determined for the outputs and inputs, respectively.

This model defines efficiency for the selected DMU as the weighted linear combination of its outputs divided by the weighted linear combination of its inputs, subject to the constraint that, for each DMU (including the one whose index z is o), the efficiency cannot exceed one. All weights are restricted to be non-negative. This formulation is non-linear; however, if one seeks to maximize the outputs while maintaining inputs constant, it is trivial to normalize the weighted inputs such that they equal one.

$$v^T x_o = 1 \tag{5}$$

This yields the following formulation.

$$\max \quad \theta = u^T y_o \tag{6}$$

$$\text{subject to} \quad u^T y_z - v^T x_z \leq 0 \quad \forall z \tag{7}$$

$$v^T x_o = 1 \tag{8}$$

$$u \geq 0 \tag{9}$$

$$v \geq 0 \tag{10}$$

In addition to the input-oriented CCR model, there is the input-oriented Banker, Charnes, and Cooper (BCC) variable returns-to-scale *DEA* model [38], which minimizes the inputs without reducing any of the outputs and assumes that the relationship between inputs and outputs involve variable returns-to-scale. Further, we consider the fact that there exist non-discretionary inputs (e.g., number of encounters representing the population) that cannot be adjusted in the optimization model. The following is known as the dual version of the input-oriented BCC *DEA* model:

$$\min \quad \theta - \eta(es_D^- + es^+) \tag{11}$$

$$\text{subject to} \quad Y\lambda - s^+ = y_o \tag{12}$$

$$X\lambda + s_D^- = \theta x_o \tag{13}$$

$$X\lambda + s_{ND}^- = x_o \tag{14}$$

$$e\lambda = 1 \tag{15}$$

$$x \geq 0, y \geq 0, \lambda \geq 0, s_D^- \geq 0, s_{ND}^- \geq 0, s^+ \geq 0 \tag{16}$$

In this formulation, there are m outputs, n inputs, and z DMUs, where technical efficiency is designated as θ; this mathematical program is run z times, once to determine the efficiency of each DMU. The index o identifies the selected DMU for which an efficiency score will be generated, and η is a small value (also known as the non-Archimedean element). Further, λ is the vector of dual multipliers, y_o is the column vector of outputs for DMU_o, x_o is the column vector of inputs for DMU_o, Y and X are matrices of outputs and inputs, respectively, e is a row vector will all elements unity, s^+ is the column vector for output slack variables, and s_D^- and s_{ND}^- are column vectors for discretionary and non-discretionary input slack variables, respectively.

As noted, this formulation partitions the inputs and input slacks into two mutually exclusive and categorically exhaustive sets, discretionary (D) and non-discretionary (ND). One can readily see that the non-discretionary input slacks are not included in the objective function, equation (11) and are not a part of the measure of efficiency evaluation that is being obtained. Further, they are not multiplied by θ in the constraint set, equation (14), so the non-discretionary input may not be reduced.

The objective function, equation (11), seeks to minimize the difference between the global efficiency and the product of the non-Archimedean element times the sum of the input excesses minus the output shortages. The constraints in equation (12) ensure that the product of the dual multipliers and output data minus the dual output slack equals to the output data for the selected DMU. The constraints given as equation (13) ensure that the product of the dual multipliers and input data plus the dual input slack (discretionary) to equal the product of the efficiency and input data for the selected DMU. The constraints, equation (14), ensure that the product of the dual multipliers and input data plus the dual input slack (non-discretionary) to equal the input data for the selected DMU. The convexity constraint, equation (15), forces the sum of the dual multipliers to equal one, which is required for a variable returns-to-scale optimization model. Finally, equation (16) lists the non-negativity constraints for the model.

Technical efficiency (or Pareto-Koopmans efficiency) is attained only if it is impossible to improve any input or output without worsening some other input or output. In other words, DMU_o is technically efficient if and only if the following two conditions are both satisfied: (i) $\theta^* = 1$, (ii) all slacks are zero (allocative efficiency is achieved). In all other cases it is possible to improve one or more of the inputs or outputs without worsening any other input or output. A DMU that achieves (i) and (ii) is then called technically efficient [38].

Although these *DEA* formulations are useful for evaluating efficiency, their use does not provide sufficient decision-support for optimizing overall system performance when inputs are fixed. Thus, in the next subsection, we revisit the *MAOM* proposed by Bastian et al. [17], which is useful for specific cases where health system decision-makers seek to balance system components that might be interpreted as a performance ratio (not necessarily efficiency). This model formulation identifies inputs that might be manipulated (re-allocated) automatically to improve system performance over multiple outputs. Some of the structure of *MAOM* can be recognized from the input-oriented BCC model.

2.2 Multi-Objective Auto-Optimization Model (*MAOM*)

Below, we introduce the notation used for sets, decision variables, data matrices of the optimization model, which is followed by the model formulation.

Optimization Model Sets

N – set of DMUs (e.g., hospitals) with $i \in N$
M – set of system output resources with $j \in M$
K – set of system input resources with $k \in K$

Optimization Model Decision Variables

δ_{ki} – adjustments to each input k by DMU i with $\delta \in \Delta$
α_{ji} – weight for output j and DMU i with $\alpha \in A$
λ_{ki} – weight for input k and DMU i with $\lambda \in \Lambda$
r – lower limit for efficiency score required for all DMUs i

Optimization Model Data Matrices

x_{ki} – input k for DMU i with $x \in X$
y_{ji} – output j for DMU i with $y \in Y$

Optimization Model Formulation

$$\max \quad Z = \sum_i \sum_j \alpha_{ji} y_{ji} \tag{17}$$

$$\text{subject to} \quad r \leq \sum_j \alpha_{ji} y_{ji} \quad \forall i \in N \tag{18}$$

$$\sum_j \alpha_{ji=v} y_{ji} - \sum_k \lambda_{ki=v}(x_{ki} + \delta_{ki}) \leq 0 \quad \forall i, v \in N \tag{19}$$

$$\sum_k \lambda_{ki=v}(x_{ki} + \delta_{ki}) = 1 \quad \forall i \in N \tag{20}$$

$$x_{ki} + \delta_{ki} \geq 0 \quad \forall k \in K, i \in N \tag{21}$$

$$\sum_i \delta_{ki} = 0 \quad \forall k \in K \tag{22}$$

$$\begin{aligned} &0 \leq r \leq 1 \\ &\alpha_{ji} \geq 0 \quad \forall i \in N, j \in M \\ &\lambda_{ki} \geq 0 \quad \forall i \in N, k \in K \\ &\delta_{ki} \quad free \quad \forall i \in N, k \in K \end{aligned} \tag{23}$$

In this optimization model, *MAOM*, the objective function, equation (17), maximizes the sum of the efficiencies for all of the DMUs, which are the weighted outputs. Equation (18) restricts the weighted outputs to be greater than or equal to a global efficiency variable $r \in [0,1]$. Equation (19) requires the sum of the weighted outputs to be less than or equal to the sum of the weighted inputs for each selected DMU $(i = v)$. This constraint makes the problem non-linear since the input weights are multiplied by the input adjustments. Equation (20) guarantees the equality of the sum of the weighted and adjusted (re-allocated) inputs to one for each DMU. Equation (21) enforces each remaining input (after adjustment) for each DMU to be greater than or equal to zero. While equation (22) requires that any input adjustments sum to zero. That is, resources cannot be increased for re-allocation. Finally, the bounds for the decision variables are given as equation (23). Extensions of this *MAOM* are possible to bound the maximum adjustment of system resources. This can increase the flexibility of the health care decision-makers and reflect management input. *MAOM* provides direct decision support for the decision maker, albeit with some limitations. The next subsection builds upon *MAOM* to construct *FMAOM*.

2.3 Fuzzy Multi-Objective Auto-Optimization Model (*FMAOM*)

One of the limitations of *MAOM* is the assumption that the global efficiency variable, r, is fixed. However, r can include possibilistic (or linguistic) uncertainty, which is difficult to describe and measure for decision-makers. In order to incorporate this into the decision-making process, we extend *MAOM* by assuming the global efficiency to be a fuzzy number and let decision-makers define the specific membership functions with respect to their strategies. These membership functions reflect the satisfaction level and risk preferences of decision-makers. For instance, one can construct a function of r, $u_i(r)$, for i^{th} DMU (i.e., hospital) as follows:

$$u_i(r) = \left[\frac{\sum_j \alpha_{ji} y_{ji} - r_{min}}{r_{max} - r_{min}} \right]^c \quad \text{where } c \geq 0, \text{ and } 0 \leq r_{min} \leq r_{max} \leq 1, \quad \forall i \in N \tag{24}$$

In equation (24), r_{min} and r_{max} are minimum and maximum levels of the technical efficiency variable, r, respectively. They are fixed parameters that are specified by the decision-maker. The parameter, c, is used to determine the risk profile of the decision-maker. Values of c between 0 and 1 correspond to risk averseness whereas values larger than 1 reflects risk seeking behavior. The risk neutral case, where c is 1 has a membership function with a monotonically increasing behavior. Figure 1 provides a visualization of the membership function, $u_i(r)$, for specific cases of $c = 0.50$, $c = 1.00$, and $c = 2.00$ that reflect the preferences of risk averse, risk neutral and risk seeker decision-makers [36,39].

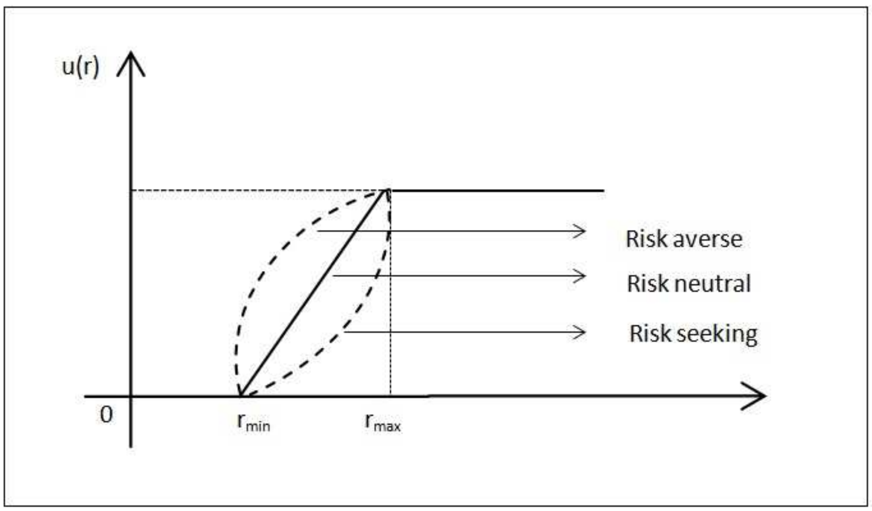

Fig. 1: Membership Function of Global Technical Efficiency [37]

We define *FMAOM* using this membership function, $u_i(r)$, and assume the technical efficiency variable, r to be a fuzzy number:

$$\tilde{r} \lesssim \sum_j \alpha_{ji} y_{ji} \quad \forall i \in N \tag{25}$$

where the symbol $\lesssim$ denotes the fuzzified aspiration level with respect to the linguistic terms of "at least" [36,37].

Using the fuzzy inequality defined in (25), *MAOM* is written as a fuzzy mathematical programming model with fuzzy constraints [40]. According to Zimmermann [33] and Werners [34], the fuzzy programming model with fuzzy constraints can be transformed into crisp model by means of the max-min operator defined by Bellman and Zadeh [35]. For models with fuzzy constraints, Werners [34] supposes that the objective function also should be fuzzy because of the fuzziness of the resources. That is, if *MAOM* is solved for r_{min} and r_{max}, respectively, then the minimum and maximum values of objective function values, Z_{min} and Z_{max} can be found. The membership function of the objective function, μ_Z, and its graph can be constructed as follows:

$$\mu_Z = 1 - \frac{Z_{max} - Z}{Z_{max} - Z_{min}}; Z_{min} \leqslant Z \leqslant Z_{max} \tag{26}$$

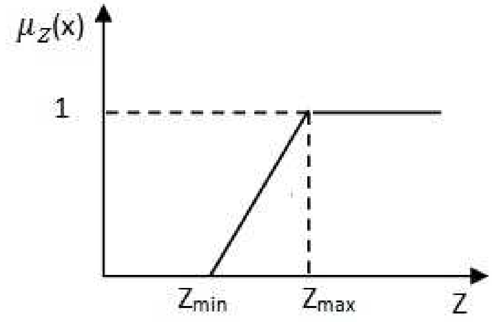

Fig. 2: Membership Function of Objective Function [39]

Here, the membership functions of the objective function and the global technical efficiencies of all the DMU's are similar to each other, and they represent the satisfaction level of decision-maker. Let's consider θ as the crisp satisfaction level for all the membership functions, then the following constraints can be obtained:

$$\begin{aligned} \theta &\leqslant \mu_Z \\ \theta &\leqslant u_i(r) \quad \forall i \in N \end{aligned} \tag{27}$$

By using the max-min operator defined by Bellman and Zadeh [35], the constraint system in (27) can be solved as follows:

$$\max_{\alpha,\sigma,\lambda \geqslant 0} \theta = \max_{\alpha,\sigma,\lambda \geqslant 0} \min[\mu_Z, u_1, u_2, \ldots, u_N] \tag{28}$$

which also corresponds to the multiobjective programming formulation of [41].

The components are now in place to present the fuzzy multi-objective auto-optimization model that maximizes the crisp satisfactory level while satisfying system constraints. After adding the other crisp system constraints of *MAOM* into the equation (28), *FMAOM* as a crisp mathematical programming problem can be written as follows.

$$\max \quad \theta \tag{29}$$

$$\text{subject to} \quad u_i(r) \geq \theta \quad \forall i \in N \tag{30}$$

$$\sum_j \alpha_{ji} y_{ji} - \sum_k \lambda_{ki=v}(x_{ki} + \delta_{ki}) \leq 0 \quad \forall i, v \in \{1, 2, \ldots, N\} \tag{31}$$

$$\sum_k \lambda_{ki=v}(x_{ki} + \delta_{ki}) = 1 \quad \forall i \in N \tag{32}$$

$$x_{ki} + \delta_{ki} \geq 0 \quad \forall k \in K, i \in N \tag{33}$$

$$\sum_i \delta_{ki} = 0; \forall k \in K \tag{34}$$

$$\begin{aligned} &\alpha_{ji} \geq 0 \quad \forall i \in N, j \in M \\ &\lambda_{ki} \geq 0 \quad \forall i \in N, k \in K \\ &\delta_{ki} \quad free \quad \forall i \in N, k \in K \end{aligned} \tag{35}$$

Note that the equations (31) – (35) are similar to the respective constraints in the *MAOM*. *FMAOM* treats r as fuzzy and defines a membership function, $u_i(r)$, as in equation (24), and a satisfaction level, θ, to model that. This provides the flexibility to reflect various decision-maker risk profiles while treating the linguistic uncertainty in the global technical efficiency variable. Different membership functions can be utilized with respect to the strategies of decision-makers. The next section provides the analysis by using c values of 0.50, 1.00, and 2.00 that reflect risk averse, risk neutral, and risk seeking decision-making preferences, respectively.

3 Computational Experiment

This section presents computational experiments that illustrate the application of the proposed methods, *MAOM* and *FMAOM*. Building on the traditional *DEA* method, the *MAOM* provides the decision-maker with recommendations for the automatic reallocation of funding and staffing, while satisfying a fixed global technical efficiency threshold, r, for all DMUs. On the other hand, the *FMAOM* treats r as a fuzzy number by the proposed membership function, $u_i(r)$ and provides recommendations based on the risk preferences of the decision-maker for any hospital.

In this computational experiment, we use annual data from 2012 that are retrieved from 54 U.S. Army hospitals in the MHS. Table 1 describes the set of health system input and output variables selected for analyzing MHS performance. These output and input parameters are typical of government facilities (e.g., Ozcan et al. [4] and O'Neill et al [42]) and are routinely used as predictors of MHS performance [15, 16].

Table 1: Description of Data Sources from the Military Health System

Variable Name	*Type*	*Description of Variables*	*Data Source*
ENROLL	Input	**Population Measure:** enrollment population supported in 2012. This input is non-discretionary.	M2
FTE	Input	**Worker Measure:** number of assigned full time equivalents in 2012.	MEPRS
COST	Input	**Cost Measure:** expenditures less graduate medical education (training) and readiness costs, inflated in two parts to 2012 dollars.	MEPRS
RWP	Output	**Inpatient Workload Measure:** aggregated hospital relative weighted product in 2012.	MEPRS
RVU	Output	**Outpatient Workload Measure:** aggregated hospital relative value unit in 2012.	MEPRS

Notes: M2 = MHS Management Analysis and Reporting Tool, a MHS data querying tool.
MEPRS = Medical Expense & Performance Reporting System, the accounting system for the MHS.

As shown in Table 1, the MHS input resources to be manipulated include $COST$, $ENROLL$, and FTE, whereas the output resources are listed as RVU and RWP. $ENROLL$ is assumed to be a non-discretionary input that cannot be re-allocated. We utilized the General Algebraic Modeling System [43] as the modeling language with optimization solvers CONOPT [44] and MINOS [45] for the non-linear components and CPLEX [46] for the traditional *DEA* analysis. Both the LP and NLP models were solved to optimality in negligible time.

First, a basic dual-BCC input-oriented, variable returns-to-scale *DEA* analysis is conducted to learn about the inefficient DMU's. Seven hospitals were found to have relative technical efficiency: H5, H8, H10, H25, H26, H41, and H45. Three of these hospitals (H5, H8, H10) were Army, two (H25, H26) were Air Force, and two (H41, H45) were Navy. The median hospital was judged to have an efficiency of 0.799, while the mean statistic was 0.784. The standard deviation of the technical efficiency scores was 0.166. For those hospitals judged to be inefficient, the most common occurring type of slack was associated with RVUs, weighted outpatient workload. Appendix A details the results of the BCC-input model along with the referent set occurrences.

Further, hospitals H5, H8, and H26 were the most common referent hospitals, having been identified as such 37, 32, and 34 times, respectively. Hospitals H5 and H8 are large Army medical centers with graduate medical education. These hospitals may be super-efficient due to the presence of lower-cost providers-in-training. Hospital H26, in contrast, is a smaller Air Force facility with comparatively little inpatient production.

In order to evaluate potential values for efficiency thresholds, a *DEA* analysis is conducted with a minimum allowed efficiency value of 0.900. As a result, 19 hospitals were found to have an efficiency value of 0.900, 10 hospitals had values in the range of 0.900 to 1.000, whereas 25 hospitals were Pareto-optimal (efficiency $= 1.000$).

Although *DEA* is a valuable tool to understand the inefficiencies and conducting manual sensitivity analysis, it does not, however, provide any direct decision support for decision-makers. Next, the *MAOM* based analysis is conducted for automatic re-allocation of inputs across all DMUs within the system to maximize overall performance. It is tested with a minimum efficiency threshold value of 0.900, and the resource changes are limited from both sides to 25% to prevent major adjustments. As a result of the automatic system input resource re-allocations, all of the hospitals other than H41 (0.961), H44 (0.960), and H48(0.900), reached technical efficiency scores of 1.000. Table 2 provides the initial and new values of resources (inputs): $COST$ and FTE. As indicated, most of the initially inefficient hospitals reached their respective limits of resource adjustments to become efficient.

Although *MAOM* results in an automatic re-allocation of input resources and increased overall system efficiency, it is based on the assumption that r is fixed and risk preferences for any hospital are the same. On the other hand, the *FMAOM* relaxes this assumption and considers the possibilistic uncertainty associated with the efficiency variable. First, the entire range of technical efficiency values is explored with values r_{min} of 0.00 and r_{max} of 1.00. Three scenarios are considered: c values of 0.50, 1.00, and 2.00 that reflect the risk averse, risk neutral, and risk seeking decision-making preferences, respectively.

For the risk averse scenario, this led to efficiency scores between 0.470 and 0.743 for 10 hospitals and an efficiency score of 1.000 for the remaining 44 hospitals without any major resource re-allocation. This can be explained by the fact that the risk averse membership function weighs the technical efficiency in a diminishing fashion with respect to the r value. For the case where $c = 1$, all hospitals were Pareto-optimal. The input resource re-allocations were minimal; there was a staffing (FTE) transfer of 1.940 from hospital H1 to H41 and expenditure ($COST$) re-allocation of 7.070 from H41 to H1. Similarly, for the risk seeking case where $c = 2.00$, all hospitals had perfect efficiencies, albeit with a higher number of resource re-allocations. Table 3 lists the resource changes, which clearly indicates that H54 has a major positive expenditure re-allocation and decrease of staffing.

Table 2: MHS Performance Results for the *MAOM* with $r = 0.90$

Hospital	*COST*	New *COST*	*FTE*	New *FTE*
H1	146.840	110.130	25.300	25.300
H2	148.340	148.332	23.180	23.180
H3	339.460	424.325	77.610	97.012
H4	186.290	186.292	40.830	40.830
H5	3581.320	3581.312	177.920	177.927
H6	654.250	658.098	123.460	123.460
H7	610.340	610.332	76.590	76.314
H8	410.750	513.437	82.010	102.512
H9	968.810	968.802	118.910	118.910
H10	153.630	192.037	50.570	51.614
H11	179.200	220.681	48.300	48.300
H12	108.170	91.655	25.120	25.120
H13	191.290	227.548	39.020	48.775
H14	982.330	988.104	79.490	90.420
H15	1716.130	2002.657	136.180	136.226
H16	323.780	331.025	65.130	65.130
H17	105.230	108.461	33.390	33.390
H18	190.810	205.557	44.690	46.037
H19	1772.890	1731.415	130.030	130.030
H20	66.870	50.152	15.160	12.812
H21	883.610	999.493	97.600	121.762
H22	331.560	272.130	52.050	52.050
H23	903.280	938.752	127.730	119.694
H24	64.210	51.986	9.480	7.120
H25	59.880	45.292	8.740	6.958
H26	53.560	40.221	11.950	11.950
H27	56.770	56.243	10.300	10.300
H28	97.890	95.780	23.460	23.460
H29	10.890	13.612	10.670	10.670
H30	1092.600	1092.592	60.290	55.721
H31	222.410	222.402	39.770	39.770
H32	233.690	247.535	38.500	38.500
H33	438.930	438.922	60.180	60.180
H34	360.870	322.050	45.490	45.490
H35	413.120	413.112	51.220	51.220
H36	38.710	40.378	40.330	40.387
H37	253.190	203.115	47.770	47.770
H38	433.740	433.732	99.370	96.404
H39	372.020	372.012	85.670	85.670
H40	219.910	168.004	24.250	18.736
H41	28.280	21.210	7.760	7.760
H42	369.580	384.443	72.790	72.790
H43	67.920	83.642	21.560	21.560
H44	57.640	43.230	16.160	16.160
H45	65.140	71.722	16.790	16.999
H46	313.060	372.129	37.320	37.320
H47	285.640	214.445	72.500	72.500
H48	64.130	48.097	11.540	11.540
H49	45.510	45.038	17.290	17.290
H50	108.000	84.874	24.440	24.440
H51	141.500	176.875	34.900	34.900
H52	1975.910	1975.902	225.850	225.850
H53	1936.320	2420.400	220.880	220.952
H54	3694.090	2770.567	238.740	179.055

Table 3: MHS Performance Results for the *FMAOM* with $r_{min} = 0.90$

Hospital	Cost Change	FTE Change
H5	-0.015	0.000
H12	-0.003	-0.002
H16	0.000	0.029
H20	-2.959	-0.007
H21	-176.312	0.005
H26	0.001	0.000
H29	0.000	0.005
H30	-0.016	0.000
H36	0.000	0.002
H38	0.000	0.005
H41	-0.002	0.000
H45	0.000	0.002
H46	-0.002	0.000
H49	0.000	0.004
H50	0.004	0.000
H53	0.131	0.000
H54	179.165	-0.042

In addition, sensitivity analysis is conducted to measure the effect of r_{min} on the optimal solution. For instance, with $r_{min} = 0.90$ and $c = 0.50$, we found that the number of hospitals with efficiency values close to 0.900 increases. There were 21 inefficient hospitals (H1, H2, H5, H11, H12, H18, H20, H26-H29, H33, H41, H44, H46-H51, H53) while the remaining 33 hospitals were Pareto-optimal. The number of inefficient hospitals was more than twice the number of such cases when r_{min} was set at 0.00. One striking observation is that 19 of these inefficient hospitals had efficiency values in the range of 0.900 to 0.903. This resembles the "all or nothing" type of decision-making and can be explained by the risk averse membership function. Resource (input) re-allocations happen between hospitals H3 and H23 in form of staffing and between hospitals H48 and H54 in form of expenditure re-allocation. The changes are in the same direction with the *MAOM* results.

A similar computational experiment is done for the risk seeking case where changes in hospital efficiencies are investigated for r_{min} values of 0.10, 0.50 and 0.90. When the value of r_{min} increases, it is found out that there are more hospitals with efficiency values either at the minimum or maximum thresholds (Figure 3). It can also be seen that the number of Pareto-efficient hospitals increase for higher level of minimum risk thresholds.

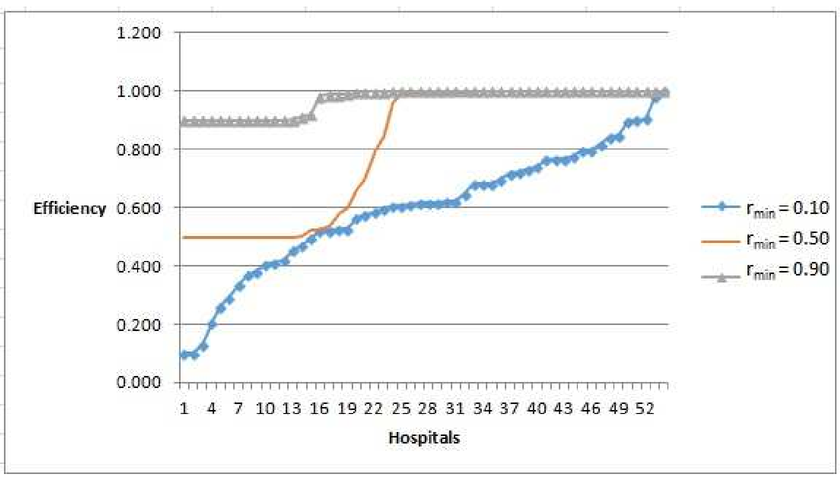

Fig. 3: Sorted Hospital Efficiency Values for *FMAOM*, $c = 2.00$ for r_{min} values of 0.10, 0.50, and 0.90

Lastly, multiple solvers are compared due to the non-linear nature of the proposed models. The congruency analysis of Bastian et al. [17] suggested there could be multiple solutions that provide the optimal objective function value. Our congruency analysis provided similar insights. The *CONOPT* [44] solver resulted with a different amount of resource changes compared to the *MINOS* [45] solution for a number of cases, however, with the same optimal objective function value. That is evidence for the existence of multiple optimal decisions. For consistency, results based upon the *CONOPT* solver are reported.

4 Concluding Remarks

This paper proposes an auto-optimization model with fuzzy constraints that can be used for automatic reallocation of resources. Specifically, it provides decision-makers of large health systems (with fixed inputs) resource re-allocation recommendations based on their risk preferences while allowing the technical efficiency variable to be fuzzy. Considering the possibilistic uncertainty of the technical efficiency variable improves the *MAOM* model by addressing the *DEA*-based limitations such as its sensitivity with respect to data.

The *MAOM* and *FMAOM* models have several implications for healthcare management and policy. Given the high degree of complexity and ambiguity in the health sector, healthcare decision-makers require analytical methods for optimizing scarce resources. In the case of fixed-budgeted organizations such as the Military Health System as well as the Veterans Health Administration and European health systems, fuzzy decision-making models are a natural choice for addressing the complexity and uncertainty. In our example, decision-makers are allowed to evaluate different risk scenarios as part of the optimization algorithm. The inclusion of these risk scenarios is (at a minimum) an important improvement to all previously-published algorithms addressing these types of problems.

First, the flexibility provided by the models allows for risk to vary by hospital. Both healthcare policy-makers and senior management can reflect risk preferences within the decision-making process. In practice, socio-demographics of the patient population served by a hospital varies significantly by location. The availability of healthcare workforce also significantly varies by location. Further, risk may change over time. For example, the implementation of the Affordable Care Act (ACA) will lead to changing patient population pools, and many hospitals will experience an increase in the number of higher risk patients to be served. The models presented here provide the opportunity for decision-makers to adjust risk accordingly with respect to their preferences. As mentioned previously, re-allocation is an important decision for management for several reasons, including the temporal nature of requirements. Both the *MAOM* and *FMAOM* models may be applied relatively easily to achieve this goal, while helping the decision-maker to consider these imminent trade-offs between cost, staffing, workload, and performance.

Illustration of the *FMAOM* with an expanded real-world data set involving 54 U.S. Army hospitals in the MHS demonstrated its versatility. In particular, we illustrated how funding and staffing input resource re-allocations can be efficiently made to maximize the overall MHS performance while considering decision-maker risk preferences. Furthermore, we provided comparisons with a basic *DEA* model and the initial *MAOM* model. Overall, the *FMAOM* model can be employed as a decision support tool for the automatic re-allocation of resources to optimize system efficiency at given risk levels. As noted by Bastian et al. [17], this automatic adjustment is advantageous as it can be extremely difficult for health system decision-makers to assign appropriate weights in the model.

The proposed models have a number of limitations. Although more flexibility is provided by means of a fuzzy global technical efficiency variable, other health system measures are left as crisp numbers. Hence, the possibilistic and probabilistic uncertainties included by these units are consciously overlooked. For future work, these units can be assumed as fuzzy or random variables as well. However, it should be noted that this will increase the number of constraints and, therefore, complexity due to the non-linearity. This approach should be used with caution in the case of improvements for single hospitals, since the reasons of inefficiencies may be different. The determination of the shapes of the membership functions and weights can also be further studied. Future work will include a case study using more hospitals, inputs and outputs. It may be worthwhile to seek ways to collect data regarding the risk preferences so that c is determined in a more accurate fashion.

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Appendix A

Table 4: MHS Performance Results for the input-oriented *DEA* model

Hospital	Type	Efficiency	Cost Slack	RWP Slack	RVU Slack	FTE Slack
H1	Army	0.543	0.000	0.000	0.000	0.000
H2	Army	0.967	0.000	0.000	0.000	0.000
H3	Army	0.913	0.000	0.000	0.000	0.000
H4	Army	0.839	26500000.000	0.000	888.477	0.000
H5	Army	1.000	0.000	0.000	0.000	0.000
H6	Army	0.703	0.000	0.000	0.000	0.000
H7	Army	0.961	0.000	0.000	0.000	0.000
H8	Army	1.000	0.000	0.000	0.000	0.000
H9	Army	0.781	69900000.000	0.000	7072.021	0.000
H10	Army	1.000	0.000	0.000	0.000	0.000
H11	Army	0.820	0.000	0.000	0.000	0.000
H12	Army	0.720	0.000	0.000	0.000	0.000
H13	Army	0.928	0.000	0.000	0.000	61.751
H14	Army	0.595	0.000	0.000	0.000	687.306
H15	Army	0.881	4380.827	0.000	0.000	1925.058
H16	Army	0.764	0.000	0.000	0.000	0.000
H17	Army	0.611	0.000	0.000	0.000	111.082
H18	Army	0.868	0.000	0.000	0.000	0.000
H19	Army	0.806	0.000	0.000	0.000	2755.739
H20	Army	0.709	0.000	0.000	0.000	6.435
H21	Army	0.644	0.000	0.000	0.000	1420.281
H22	Army	0.831	0.000	0.000	0.000	0.000
H23	Army	0.704	0.000	0.000	0.000	7.937
H24	Air Force	0.999	1227151.161	0.000	36.300	0.000
H25	Air Force	1.000	0.000	0.000	0.000	0.000
H26	Air Force	1.000	0.000	0.000	0.000	0.000
H27	Air Force	1.004	9237625.836	0.000	30.464	0.000
H28	Air Force	0.942	0.000	0.000	0.000	300.770
H29	Air Force	0.965	195.115	0.000	0.000	83.978
H30	Air Force	0.729	0.000	0.000	0.000	153.201
H31	Air Force	0.911	0.000	0.000	0.000	439.055
H32	Air Force	0.854	0.000	0.000	0.000	0.000
H33	Air Force	0.797	0.000	0.000	0.000	0.000
H34	Air Force	0.872	0.000	0.000	0.000	184.722
H35	Air Force	0.909	0.000	0.000	0.000	0.000
H36	Navy	0.369	0.000	127.415	0.000	157.389
H37	Navy	0.627	0.000	0.000	0.000	428.012
H38	Navy	0.781	0.000	0.000	0.000	0.000
H39	Navy	0.645	0.000	0.000	0.000	1406.978
H40	Navy	0.768	0.000	0.000	0.000	322.167
H41	Navy	1.000	0.000	0.000	0.000	0.000
H42	Navy	0.582	0.000	0.000	0.000	187.340
H43	Navy	0.724	12.749	0.000	0.000	483.079
H44	Navy	0.542	581782.067	0.000	30.180	0.000
H45	Navy	1.000	0.000	0.000	0.000	0.000
H46	Navy	0.831	0.000	0.000	0.000	594.561
H47	Navy	0.416	0.000	0.000	0.000	1029.131
H48	Navy	0.712	27.920	0.000	0.000	49.308
H49	Navy	0.549	0.000	0.000	0.000	2446.434
H50	Navy	0.800	0.000	0.000	0.000	242.273
H51	Navy	0.517	0.000	0.000	0.000	793.524
H52	Navy	0.659	2700.164	0.000	0.000	5257.315
H53	Navy	0.640	2607.059	0.000	0.000	3038.174
H54	Navy	0.629	39400000.000	0.000	9135.797	0.000

Table 5: MHS Referent Hospitals for the input-oriented *DEA* model

Hospital	H5	H8	H10	H25	H26	H41	H45
H1	1	1	0	0	1	0	1
H2	1	0	0	0	1	0	1
H3	1	1	1	0	0	0	0
H4	1	0	0	0	1	0	0
H5	1	0	0	0	0	0	0
H6	1	1	1	0	0	0	1
H7	1	1	1	0	0	0	0
H8	0	1	0	0	0	0	0
H9	1	0	0	0	1	0	0
H10	0	0	1	0	0	0	0
H11	1	1	1	0	0	0	1
H12	1	0	0	1	1	0	0
H13	0	1	0	0	1	0	1
H14	1	1	0	0	1	0	0
H15	1	1	0	0	0	0	0
H16	1	1	1	0	0	0	1
H17	0	1	0	0	1	0	1
H18	1	1	1	0	0	0	1
H19	1	1	0	0	1	0	0
H20	1	0	0	1	1	0	0
H21	1	1	0	0	1	0	0
H22	1	1	1	0	0	0	1
H23	1	1	0	0	1	0	0
H24	0	0	0	1	1	0	0
H25	0	0	0	1	0	0	0
H26	0	0	0	0	1	0	0
H27	0	0	0	1	1	0	0
H28	0	1	0	0	1	0	1
H29	0	0	0	1	0	1	0
H30	1	1	0	0	1	0	0
H31	1	1	0	0	1	0	0
H32	1	1	0	0	1	0	1
H33	1	0	1	0	0	0	1
H34	1	1	0	0	1	0	0
H35	1	0	0	0	1	0	1
H36	1	1	0	0	1	0	0
H37	1	1	0	0	1	0	0
H38	1	1	0	0	1	0	1
H39	1	1	0	0	1	0	0
H40	1	0	0	1	1	0	0
H41	0	0	0	0	0	1	0
H42	1	1	0	0	1	0	0
H43	0	0	0	0	1	0	1
H44	0	0	0	1	1	0	0
H45	0	0	0	0	1	0	1
H46	1	1	0	0	1	0	0
H47	1	1	0	0	1	0	0
H48	0	0	0	1	0	1	0
H49	0	0	0	1	0	1	0
H50	0	1	0	0	1	0	1
H51	1	1	0	0	1	0	0
H52	1	1	0	0	0	0	0
H53	1	1	0	0	0	0	0
H54	1	0	1	0	0	0	0
Total	37	32	10	10	34	4	17