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Stochastic Multi-Objective Auto-Optimization for Resource Allocation Decision-Making in Fixed-Input Health Systems

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Abstract The management of hospitals within fixed-input health systems such as the U.S. Military Health System (MHS) can be challenging due to the large number of hospitals, as well as the uncertainty in input resources and achievable outputs. This paper introduces a stochastic multi-objective auto-optimization model (*SMAOM*) for resource allocation decision-making in fixed-input health systems. The model can automatically identify where to re-allocate system input resources at the hospital level in order to optimize overall system performance, while considering uncertainty in the model parameters. The model is applied to 128 hospitals in the three services (Air Force, Army, and Navy) in the MHS using hospital-level data from 2009 – 2013. The results are compared to the traditional input-oriented variable returns-to-scale Data Envelopment Analysis (*DEA*) model. The application of *SMAOM* to the MHS increases the expected system-wide technical efficiency by 18% over the *DEA* model while also accounting for uncertainty of health system inputs and outputs. The developed method is useful for decision-makers in the Defense Health Agency (DHA), who have a strategic level objective of integrating clinical and business processes through better sharing of resources across the MHS and through system-wide standardization across the services. It is also less sensitive to data outliers or sampling errors than traditional DEA methods.

Keywords Multi-objective optimization · stochastic programming · resource allocation · performance measurement · productivity analysis · health systems · military medicine

1 Introduction

Healthcare systems in the United States are under increasing pressure to provide world-class healthcare that is safe, effective, patient-centered, timely, efficient, and equitable [14]. Healthcare systems are expected to deliver high-level care while keeping their associated healthcare costs at the same or lower level. This poses a great challenge given the expense of new healthcare technology, specialized procedures, customizable medicine, and a highly-skilled labor force requirements.

The U.S. Military Health System (MHS) is a global system delivering health services with medical readiness at the center of its mission. The MHS consists of three military departments comprising the three uniformed services

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(Air Force, Army, and Navy) and the Defense Health Agency (DHA). The DHA has the strategic goal to support the delivery of integrated, affordable, and high-quality health services to beneficiaries of the MHS. The DHA is responsible for driving greater integration of clinical and business processes across the MHS, implementing shared services with common measurement outcomes, enabling rapid adoption of proven practices, helping reduce unwanted variation, and improving the coordinating of care across time and treatment venues [15].

The MHS is unique in that it recruits and trains its own medical staff, has a generally physically fit patient population, and is a closed, single payer system. It is an integral component of our U.S. military fighting force – ensuring a medically ready force and a ready medical force to respond to the full spectrum of military operations. Within the integrated MHS, there are 9.5 million beneficiaries, 699 medical treatment facilities (broken into hospitals, medical clinics, and dental clinics), and 380,000 medical professionals (combining military, civilian and contractors) working in concert with the TRICARE network of providers. The MHS has an education and training system that includes an accredited medical school, graduate programs, and enlisted and officer training platforms. Further, it has comprehensive, cutting-edge medical research and development programs [15].

As a large, centrally-funded and controlled health system, the MHS is challenged to provide healthcare delivery and health services at high quality with specified workload levels with a fixed amount of input resources (e.g., staffing, funding, etc.). Although the MHS shares some of the same challenges as large civilian US-based health systems when it comes to delivering world-class healthcare, its unique mission comes with its own set of healthcare challenges. At the surface, the MHS is charged with delivering quality healthcare to a diverse population. At the core, however, that charge includes maintaining peacetime healthcare delivery capacity while ensuring the deployment readiness of the active force. The MHS is also charged with deploying, establishing and running forward deployed healthcare facilities. Further complicating the delivery of quality care is the transient nature of healthcare providers either due to deployments or routine personnel moves between hospitals, clinics, and field units. Given these challenges, MHS leaders need to understand and objectively evaluate how their hospitals and clinics perform.

In order to optimize system performance across the MHS, the fixed-input system resources can be effectively and efficiently allocated across the military hospital network while maintaining health system outputs. As a result, senior leaders in the DHA and other large fixed-input health systems (such as the U.S. Veterans Health Administration) seek performance measurement methods that systematically update with these strategic and policy challenges. Moreover, there is tremendous uncertainty involved with both the input and output health system resources, which poses yet another obstacle in meeting certain target performance levels across the system in an efficient manner.

In response to these challenges, this paper proposes a stochastic multi-objective auto-optimization model for resource allocation decision-making in fixed-input health systems. This performance measurement method can help health system decision-makers automatically re-allocate input resources across hospitals for different levels of resource uncertainty, while performing sensitivity analysis needed to optimize overall system performance.

1.1 Literature Review

The focus of performance measurement in health systems is on the relationship among inputs (such as staffing, funding, etc.) and outputs (such as outpatient workload, quality metrics, etc.). Hollingsworth [27] and Hollingsworth et al. [28] provided comprehensive discussions about the non-parametric and parametric methods that are used for efficiency measurement in health care. Charnes et al. [6] proposed the use of data envelopment analysis (DEA) for such productivity analysis. Ozcan et al. [48] evaluated trends in hospital efficiency using longitudinal data. Fulton et al. [22] proposed regression-based military hospital cost models that included DEA efficiency scores in addition to the variables of quality, access, and satisfaction. Nayar and Ozcan [44] provided a DEA comparison of Virginia hospitals in terms of efficiency and quality. Ferrier and Trivitt [20] also utilized a double DEA approach in which they controlled for quality and efficiency. Jacobs [32] compared DEA and stochastic frontier analysis (SFA) in terms of their ability to rank efficient hospitals in United Kingdom. Overall, these methods provide performance measurement and sensitivity analysis, but not direct decision support for senior managers in health systems.

The available decision-making methods range from goal programming to decision support systems. Eichler et al. [18] provided an overview of the cost-effectiveness analysis for healthcare resource allocation decision-making. The goal programming model of Kwak et al. [37] allows decision-makers to make strategic allocation decisions with limited human resources in a health system subject to the patient satisfaction constraints. Kwak et al. [36] used the analytic hierarchy process to identify and prioritize the goal levels within a similar multi-criteria mathematical program. Aktas et al. [1] proposed a management-oriented decision support model which employed a Bayesian belief network that models the causality among the key variables of system efficiency. Fulton et al. [21] used DEA and SFA to identify the cost drivers for performance-based resource allocation, which later are used as part of the direct decision support methods for the MHS. Bastian et al. [3] proposed a multi-objective auto-optimization model that is built on the sensitivity analysis of DEA models.

These aforementioned models that utilize DEA assume that inputs and outputs are deterministic. However, the non-parametric nature of DEA relies on comparison with extreme observations. Therefore, the results can be extremely sensitive to sampling error and errors in variables. For instance, outliers such as superefficient entities may skew the results. Hence, a key to the success of the DEA approach is the accurate measurement of all factors, including inputs and outputs. However, inputs and outputs can be volatile and complex to measure accurately in certain environments. SFA-based methods consider uncertainty, primarily due to measurement errors and missing variables, but not necessarily with respect to the decision environment. O'Donnell [45] highlighted the importance of considering uncertainty while evaluating efficiency in a stochastic decision environment. Seiford [49] and Lovell [42] argued for remaining skepticism of the managerial and policy implications drawn from DEA unless the uncertainty of input and output data parameters is addressed.

There is a body of work about the statistical properties of DEA estimators, for example Simar [54] and Banker [2]. Grosskopf [25] presented a survey focusing on the statistical approaches for frontier estimation and efficiency evaluation, whereas Cooper et al. [8] discussed sensitivity analyses of the effects occurring from data variations. Simar and Wilson [55] discussed approaches such as bootstrapping in order to consider sampling errors. Another option is to use Bayesian methods to account for sampling error in DEA [56]. Kuosmanen and Post [35] generalized the notion of economic efficiency to derive necessary and sufficient first-order stochastic dominance efficiency conditions. Cooper et al. [13] proposed a range adjusted measure of inefficiency for use with additive models and discussed such models and measures in DEA.

An overview of the key approaches to handling uncertainty in DEA models is provided by Dyson and Shale [17]. DEA approaches that attempt to deal with randomness can be dated back to the use of expected values by Sengupta [53]. Sengupta [50–52] also introduced a number of mathematical programming formulations for stochastic DEA. The employment of chance-constrained programming formulations within stochastic DEA models has also become widespread, so that the stochastic variations in data are accommodated. Chance-constrained DEA (CC-DEA) models incorporate random disturbances in the data in inputs and outputs given that the respective probability distributions are known. Land et al. [38] utilized chance-constrained formulations to develop efficient frontiers that envelop a given set of observations with a pre-determined probability.

Olesen and Petersen [47] proposed a CC-DEA model that uses piece-wise linear envelopments of confidence regions for observed stochastic multiple inputs and outputs. Olesen [46] presented a comparison of these two models. Cooper et al. [10] proposed the use of satisficing concepts in DEA that has a marginal chance constraint on each decision-making unit (DMU). Cooper et al. [11] discussed stochastic characterizations of efficiency and dominance in such DEA models. Land et al. [39] provided an empirical comparison of the productive efficiency of a set of Western European market economies and a set of Eastern European planned economies using the CC-DEA. Huang and Li [29] and Li [40] generalized conventional DEA models by incorporating random disturbances into input and output data and analyzed stochastic variable returns-to-scale models using the idea of stochastic supporting hyper-planes. Huang and Li [30] developed CC-DEA models by defining the efficiency measures via joint probabilistic comparisons of inputs and outputs with other DMUs.

Notable stochastic DEA approaches also include Monte Carlo DEA (MC-DEA) which is based on using a Monte Carlo simulation for the input/output values of each DMU from their statistical distribution to determine the distribution of each DMU's relative technical efficiency [33]. A number of fuzzy data envelopment analysis (F-DEA) studies have also been proposed to deal with imprecision of input and output data; see the overview of Hatami-Marbini et al. [26]. FDEA approaches can be categorized into six groups: the tolerance approach, the α-level based approach, the fuzzy ranking approach, the possibility approach, the fuzzy arithmetic, and the fuzzy random/type-2 fuzzy set [23]. When some outputs and inputs are unknown decision variables such as bounded data, ordinal data, and ratio bounded data, the DEA model becomes a non-linear programming problem and is called imprecise DEA (IDEA) [57]. Cooper et al. [12] and Kim et al. [34] utilized scale transformations and variable alternations to transform it into a linear program. It should be noted that IDEA discusses imprecise data in the context of DEA multipliers or weights, and is different from the stochastic or CC-DEA approach where imprecise data are estimated with probabilities and the focus is on the outputs/inputs.

The use of stochastic DEA models in healthcare is relatively new. Mitropoulos et al. [43] proposed a combined application of a CC-DEA model that is integrated with a stochastic mechanism from Bayesian techniques with an application in the Greek health system. Jimenez et al. [33] used MC-DEA to evaluate the relative technical efficiency of small health care areas in probabilistic terms with respect to both mental health care, as well as the efficiency of the entire system. Lin et al. [41] utilized a multi-objective simulation optimization using DEA and genetic algorithms to determine optimal resource levels in surgical services. An example to a F-DEA application in health system efficiency evaluation is the fuzzy resource allocation model of Ekin et al. [19].

1.2 Motivation and Overview

In the previous literature, the evaluation of slack and reduced costs in traditional stochastic, multi-objective mathematical programming approaches does not provide sufficient decision support for the re-allocation of system resources. For large, centrally-funded and controlled health systems such as the MHS, it is critical to efficiently and effectively re-allocate system resources since total funding and staffing is fixed. Nevertheless, decision-makers are under pressure to sustain health system output objectives while cost and demand for health services increase.

Bastian et al. [3] proposed an optimization model linking traditional DEA and multi-objective optimization to help the decision-maker balance competing output objectives automatically. Their multi-objective auto-optimization model provides DHA decision-makers with the required decision support for automatic re-allocation of input resources within a fixed-input system. In particular, the model adjusts resources automatically across all hospitals in the system to achieve maximum system performance while achieving a minimum level of technical efficiency for each hospital. One limitation of their approach, however, is the assumption of deterministic health system input and output resources. This paper overcomes this limitation by extending the work to account for uncertainty in the data parameters using stochastic programming.

This paper is structured as follows. In Section 2, we present the materials and methods, while in Section 3 we perform a computational experiment of the proposed model. Section 4 provides concluding remarks and areas of future work.

2 Materials and Methods

This section first reviews the basic concepts of *DEA* and then revisits the multi-objective auto-optimization model (*MAOM*) of Bastian et al. [3], and extends it to introduce the proposed stochastic multi-objective auto-optimization model (*SMAOM*). It should be noted that the *MAOM* is a resource allocation-based optimization model where decision-makers can perform sensitivity analysis and re-allocate system input resources automatically within a fixed-input health system. Whereas the *SMAOM* introduces stochastic programming into this model to account for uncertainty in the data parameters. Detailed descriptions of all models are in the following subsections.

2.1 Data Envelopment Analysis (*DEA*)

DEA is a deterministic, non-parametric approach that uses linear programming (LP) for the assessment of efficiency. In the constant returns-to-scale Charnes, Cooper, and Rhodes (CCR) *DEA* model [7], efficiency is defined as a linear combination of the weighted outputs divided by a linear combination of the weighted inputs. In *DEA*, assume that an organization (e.g., health system) wishes to assess the relative efficiencies of some set of comparable sub-units (e.g, hospitals), so called Decision Making Units (DMUs). For each DMU, there is a vector of associated inputs and outputs of managerial interest [9]. Often, the decision-maker is interested in minimizing the inputs without reducing any of the outputs (known as input-oriented). One may write the primal LP formulation of the input-oriented CCR constant returns-to-scale *DEA* model as follows:

$$\max \quad \theta = u^T y_o \tag{1}$$

$$\text{subject to} \quad u^T y_i - v^T x_i \leq 0 \quad \forall i \in z \tag{2}$$

$$v^T x_o = 1 \tag{3}$$

$$u \geq 0, v \geq 0 \tag{4}$$

In this LP formulation, there is a vector of outputs (y), a vector of inputs (x), and z DMUs. The index o identifies the selected DMU for which an efficiency score will be generated. This LP model optimizes efficiency, designated as θ, by maximizing the outputs while maintaining constant inputs. The sum of the weighted outputs must be no greater than the sum of the weighted inputs, and the sum of the weighted inputs must equal one and the sum. The model is run z times for the total number of DMUs, once to determine the efficiency of each DMU. The vector of input weights (v) and vector of output weights (u) are the decision variables to be determined.

In addition to the input-oriented CCR constant returns-to-scale *DEA* model, there is the input-oriented Banker, Charnes, and Cooper (BCC) variable returns-to-scale *DEA* model [9]. The BCC *DEA* model minimizes the inputs without reducing any of the outputs while assuming that the relationship between inputs and outputs involve variable returns-to-scale. We should also consider the fact that there often exist non-discretionary inputs (e.g., patient enrollment population) that cannot be adjusted in the LP model. Hence, the following is the dual LP formulation of the input-oriented BCC variable returns-to-scale *DEA* model:

$$\min \quad \theta - \eta(es_D^- + es^+) \tag{5}$$

$$\text{subject to} \quad Y\lambda - s^+ = y_o \tag{6}$$

$$X\lambda + s_D^- = \theta x_o \tag{7}$$

$$X\lambda + s_{ND}^- = x_o \tag{8}$$

$$e\lambda = 1 \tag{9}$$

$$x \geq 0, y \geq 0, \lambda \geq 0, s_D^- \geq 0, s_{ND}^- \geq 0, s^+ \geq 0 \tag{10}$$

In this LP formulation, the inputs and input slacks are partitioned into two mutually exclusive and categorically exhaustive sets, discretionary (*D*) and non-discretionary (*ND*). It should be noted that the non-discretionary input slacks are not included in the objective function (5) and are not a part of the measure of efficiency evaluation that is being obtained. Also, they are not multiplied by θ in the constraint set (8), so the non-discretionary input may not be reduced. Further, there are m outputs, n inputs, and z DMUs, where technical efficiency is designated as θ; this mathematical program is run z times, once to determine the efficiency of each DMU. The index o identifies the selected DMU for which an efficiency score will be generated, and η is a small value (also known as the non-Archimedean element). Further, λ is the vector of dual multipliers, y_o is the column vector of outputs for DMU_o, x_o is the column vector of inputs for DMU_o, Y and X are matrices of outputs and inputs, respectively, e is a row vector will all elements unity, s^+ is the column vector for output slack variables, and s_D^- and s_{ND}^- are column vectors for the discretionary and non-discretionary input slack variables, respectively.

The objective function, equation (5), seeks to minimize the difference between the global efficiency and the product of the non-Archimedean element times the sum of the input excesses and the output shortages. The constraints in equation (6) ensure that the product of the dual multipliers and output data minus the dual output slack equals to the output data for the selected DMU. The constraints given as equation (7) ensure that the product of the dual multipliers and input data plus the dual input slack (discretionary) to equal the product of the efficiency and input data for the selected DMU. The constraints, equation (8), ensure that the product of the dual multipliers and input data plus the dual input slack (non-discretionary) to equal the input data for the selected DMU. The convexity constraint, equation (9), forces the sum of the dual multipliers to equal one, which is required for a variable returns-to-scale optimization model. Equation (10) lists the non-negativity constraints for the model.

Although these *DEA* formulations are useful for evaluating efficiency, their use does not provide sufficient decision support for optimizing overall system performance when input resources are fixed. Thus, in the next subsection, we revisit the *MAOM* proposed by Bastian et al. [3], which is useful for specific cases where health system decision-makers seek to identify system inputs that might be re-allocated automatically to improve overall system performance over multiple output objectives.

2.2 Multi-Objective Auto-Optimization Model (*MAOM*)

We now introduce the notation used for the sets, parameters, and decision variables of the optimization model, which is followed by the *MAOM* formulation.

2.2.1 MAOM Optimization Model Sets

N – set of DMUs with $i \in N$
M – set of system output resources with $j \in M$
K – set of system input resources with $k \in K$

2.2.2 MAOM Optimization Model Parameters

x_{ki} – input k for DMU i with $x \in X$
y_{ji} – output j for DMU i with $y \in Y$
r – minimum efficiency score required for all DMUs
γ – percentage of allowable input resource change to prevent major adjustments

2.2.3 MAOM Optimization Model Decision Variables

δ_{ki} – adjustments to each input k by DMU i with $\delta \in \Delta$
α_{ji} – weight for output j and DMU i with $\alpha \in A$
λ_{ki} – weight for input k and DMU i with $\lambda \in \Lambda$

2.2.4 MAOM Optimization Model Formulation

$$\max \quad Z = \sum_i \sum_j \alpha_{ji} y_{ji} \tag{11}$$

$$\text{subject to} \quad r \leq \sum_j \alpha_{ji} y_{ji} \quad \forall i \in N \tag{12}$$

$$\sum_j \alpha_{ji=v} y_{ji} - \sum_k \lambda_{ki=v}(x_{ki} + \delta_{ki}) \leq 0 \quad \forall i, v \in N \tag{13}$$

$$\sum_k \lambda_{ki}(x_{ki} + \delta_{ki}) = 1 \quad \forall i \in N \tag{14}$$

$$x_{ki} + \delta_{ki} \geq 0 \quad \forall k \in K, i \in N \tag{15}$$

$$\sum_i \delta_{ki} = 0 \quad \forall k \in K \tag{16}$$

$$\delta_{ki} \geq -\gamma x_{ki} \quad \forall k \in K, i \in N \tag{17}$$

$$\delta_{ki} \leq \gamma x_{ki} \quad \forall k \in K, i \in N \tag{18}$$

$$0 \leq r \leq 1 \tag{19}$$

$$\begin{aligned} &\alpha_{ji} \geq 0 \quad \forall i \in N, j \in M \\ &\lambda_{ki} \geq 0 \quad \forall i \in N, k \in K \\ &\delta_{ki} \quad free \quad \forall i \in N, k \in K \end{aligned} \tag{20}$$

In the *MAOM* formulation, the objective function in (11) seeks to maximize the sum of the efficiencies for all of the DMUs, which are the weighted outputs. Constraints (12) restrict the weighted outputs to be greater than or equal to the minimum efficiency score required for all DMUs. Constraints (13) require the sum of the weighted outputs to be less than or equal to the sum of the weighted inputs for each selected DMU $(i = v)$. This constraint makes the problem non-linear since the input weights are multiplied by the input adjustments. Constraints (14) guarantee the equality of the sum of the weighted and adjusted (re-allocated) inputs to one for each DMU. Constraints (15) enforce each remaining input (after adjustment) for each DMU to be greater than or equal to zero. Constraints (16) require that any input adjustments sum to zero. That is, resources cannot be increased for re-allocation. Constraints (17) prevent rapid decreases of inputs, while constraints (18) prevent rapid increases of inputs. Constraint (19) ensures that the minimum efficiency score $r \in [0,1]$ for all DMUs. Finally, the bounds for the decision variables are given in constraints (20).

The next sub-section builds upon the *MAOM* to construct the *SMAOM*.

2.3 Stochastic Multi-Objective Auto-Optimization Model (*SMAOM*)

The stochastic multi-objective auto-optimization model (*SMAOM*) uses a scenario-based Monte Carlo simulation approach slightly modified from Bastian et al. [5]. Here, our optimization objective is a maximization problem, whereas Bastian et al. [5] considered a minimization problem. In this approach, we approximate the expected optimal objective value and the optimal solution of the *MAOM* with uncertain input and output resources:

$$\max_{x \in X} E[f(x, \xi)]. \tag{21}$$

Using longitudinal data from the organization, we fit separate probability distributions based on the panels (multiple time periods) for each DMU, system input resource type, and system output resource type (i.e., $N * M * K$ distributions). We then stochastically generate a scenario by simultaneous random sampling from each of the different fitted probability distributions; this procedure equates to one Monte Carlo simulation random draw. In other words, each scenario is stochastically generated by randomly sampling with replacement from probability distributions for each DMU, input resource, and output resource. In order to perform Monte Carlo simulation, we randomly generate S scenarios where each scenario corresponds to one realization of the random vector ξ. As a result, we have S deterministic *MAOM* problems with the forms of:

$$\max_{x \in X} f(x, \xi_s) \quad \text{where} \quad s = 1, 2, .., S \tag{22}$$

If we let $z_s = \max_{x \in X} f(x, \xi_s)$ be the optimal objective value for scenario s and $x_s = arg \max_{x \in X} f(x, \xi_s)$ be the optimal solution for scenario s, then we obtain the expected optimal objective value $\bar{z}$ and the expected optimal solution $\bar{x}$, where the Monte Carlo estimates are:

$$\bar{z} = \frac{1}{S} \sum_{s=1}^{S} z_s \tag{23}$$

$$\bar{x} = \frac{1}{S} \sum_{s=1}^{S} x_s \tag{24}$$

Therefore, we solve the deterministic *MAOM S* times (once for each scenario) with stochastically generated input and output resources, where the final objective value and optimal solution is the average over the scenarios (i.e., Monte Carlo estimates). We also compute the standard deviation of the objective value and solution as follows:

$$V_z = \sqrt{\frac{1}{S-1} \sum_{s=1}^{S} (z_s - \bar{z})^2} \tag{25}$$

$$V_x = \sqrt{\frac{1}{S-1} \sum_{s=1}^{S} (x_s - \bar{x})^2} \tag{26}$$

Given this basic understanding of our scenario-based Monte Carlo simulation approach for solving stochastic programming models, we now introduce the notation used for the sets, parameters, and decision variables of the *SMAOM* formulation.

2.3.1 SMAOM Optimization Model Sets

N – set of DMUs with $i \in N$
$\widetilde{M}$ – set of stochastic system output resources with $j \in \widetilde{M}$
$\widetilde{K}$ – set of stochastic system input resources with $k \in \widetilde{K}$

2.3.2 SMAOM Optimization Model Parameters

$\widetilde{x}_{ki}$ – stochastic input k for DMU i with $\tilde{x} \in \widetilde{X}$
$\widetilde{y}_{ji}$ – stochastic output j for DMU i with $\tilde{y} \in \widetilde{Y}$
r_i – minimum efficiency score required for DMU i
r_{min} – lower bound on minimum efficiency score r_i
r_{max} – upper bound on minimum efficiency score r_i
γ – percentage of allowable input resource change to prevent major adjustments

2.3.3 SMAOM Optimization Model Decision Variables

δ_{ki} – adjustments to each input k by DMU i with $\delta \in \Delta$
α_{ji} – weight for output j and DMU i with $\alpha \in A$
λ_{ki} – weight for input k and DMU i with $\lambda \in \Lambda$

2.3.4 SMAOM Optimization Model Formulation

For $s = 1, 2, .., S$:

$$\max \quad z_s = \sum_i \sum_j \alpha_{ji} \widetilde{y}_{ji} \tag{27}$$

$$\text{subject to} \quad r_i \leq \sum_j \alpha_{ji} \widetilde{y}_{ji} \quad \forall i \in N \tag{28}$$

$$\sum_j \alpha_{ji=v} \widetilde{y}_{ji} - \sum_k \lambda_{ki=v} (\widetilde{x}_{ki} + \delta_{ki}) \leq 0 \quad \forall i, v \in N \tag{29}$$

$$\sum_k \lambda_{ki}(\widetilde{x}_{ki}+\delta_{ki}) = 1 \quad \forall i \in N \tag{30}$$

$$\widetilde{x}_{ki}+\delta_{ki} \geq 0 \quad \forall k \in \widetilde{K}, i \in N \tag{31}$$

$$\sum_i \delta_{ki} = 0 \quad \forall k \in \widetilde{K} \tag{32}$$

$$\delta_{ki} \geq -\gamma\widetilde{x}_{ki} \quad \forall k \in \widetilde{K}, i \in N \tag{33}$$

$$\delta_{ki} \leq \gamma\widetilde{x}_{ki} \quad \forall k \in \widetilde{K}, i \in N \tag{34}$$

$$r_{min} \leq r_i \leq r_{max} \quad \forall i \in N \tag{35}$$

$$\begin{aligned} &\alpha_{ji} \geq 0 \quad \forall i \in N, j \in \widetilde{M} \\ &\lambda_{ki} \geq 0 \quad \forall i \in N, k \in \widetilde{K} \\ &\delta_{ki} \quad free \quad \forall i \in N, k \in \widetilde{K} \end{aligned} \tag{36}$$

In this *SMAOM* formulation, for each scenario s we let $\delta_{kis} = \delta_{ki}$, $\alpha_{jis} = \alpha_{ji}$, and $\lambda_{kis} = \lambda_{ki}$, and the efficiency score of each DMU is $\theta_{is} = \sum_j \alpha_{ji}\widetilde{y}_{ji}$. After solving the optimization model for each stochastically generated scenario, we compute the expected objective function value as $\bar{z} = \frac{1}{S}\sum_{s=1}^{S} z_s$, the expected adjustment to each input k by DMU i as $\bar{\delta_{ki}} = \frac{1}{S}\sum_{s=1}^{S}\delta_{kis}$, the expected efficiency of each DMU i as $\bar{\theta}_i = \frac{1}{S}\sum_{s=1}^{S}\theta_{is}$, and the standard deviation of the efficiency of each DMU i as $V_{\theta_i} = \sqrt{\frac{1}{S-1}\sum_{s=1}^{S}(\theta_{is}-\bar{\theta}_i)^2}$.

3 Computational Experiment

This section presents a computational experiment to illustrate the application of the proposed methods. In particular, the traditional input-oriented BCC *DEA* model is compared to the *MAOM* and *SMAOM* for 128 hospitals (i.e., DMUs) from the MHS. Annual hospital-level data from $2009-2013$ are used to compute the empirical mean (and standard deviation) value for each of the inputs and outputs of interest. These averaged values of the inputs and outputs are used for the *DEA* and *MAOM* analyses. For the *SMAOM* analysis, the stochastic inputs and outputs for each DMU are simultaneously generated in each scenario s using fitted probability distributions. Note that the data set includes measures for 33 Army hospitals, 69 Air Force clinics, and 26 Navy hospitals. Table 1 describes the set of MHS input and output variables used for optimizing overall health system performance.

Table 1: Description of Data Sources from the MHS, 2009-2013

Variable Name	*Type*	*Description of Variables*
ENROLL	Input	**Population Measure:** enrollment population supported. This input is non-discretionary.
FTE	Input	**Staffing Measure:** number of available full time equivalents.
EXP	Input	**Funding Measure:** expenditures less graduate medical education and readiness costs.
RVU	Output	**Outpatient Workload Measure:** aggregated hospital relative value units.
QUAL	Output	**Quality Measure:** annual average Healthcare Effectiveness Data Information Set (HEDIS) score.

These data elements were one-time extracted from the MHS Management Analysis and Report Tool (M2), a MHS data querying tool. Note that these specified data parameters are typical of government hospitals [48] and are often used as variables in MHS performance measurement and evaluation studies [21,22,3,19,4]. As shown in Table 1, the MHS input resources to be manipulated include EXP, *ENROLL*, and FTE, whereas the output resources are listed as RVU and *QUAL*. *ENROLL* is assumed to be a non-discretionary input that cannot be re-allocated, yet it still affects hospital efficiency. The output variable, RVU, is a common output measure representing weighted outpatient workload. The *QUAL* variable is the percent of HEDIS measures for which an organization was compliant in a given year. HEDIS is a set of quality performance output measures promulgated by the National Committee for Quality Assurance, and the measures reflect outcomes (outputs).

We utilized the General Algebraic Modeling System (GAMS) [24] as the modeling language with optimization solver CONOPT [16] for the nonlinear programming models (*MAOM*, *SMAOM*); this feasible path solver is based on the generalized reduced gradient (GRG) algorithm. The optimization solver CPLEX [31] was used for the *DEA* analysis. Both the *DEA* and *MAOM* models were solved to optimality in negligible time, while the *SMAOM* model took longer time depending on the number of scenarios.

3.1 Data Envelopment Analysis (*DEA*) Results

As part of the input-oriented BCC *DEA* analysis, technical efficiency for each of the 128 military hospitals was computed using the average value (from 2009 – 2013) of the input and output resources. Figure 1 shows the distribution of *DEA* efficiency scores. The mean *DEA* efficiency score is 0.79, with a standard deviation of 0.16. Among the 128 military hospitals, 29 hospitals (23%) were found to be technically efficient (i.e., have an efficiency score of 1). Note that 15 of these hospitals were Army, six were Air Force, and eight were Navy. The *DEA* efficiency scores were approximately normally distributed, except for the most efficient hospital group (38 hospitals) that achieved technical efficiency greater than equal to 0.95.

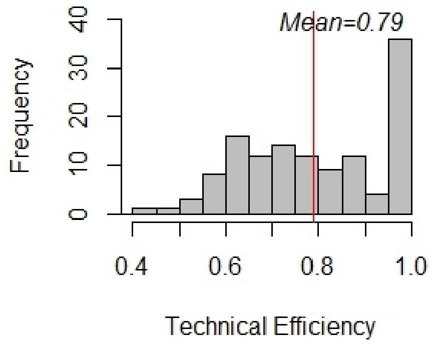


Fig. 1: Distribution of *DEA* Efficiency Scores

Table 2 summarizes the slack of individual input factors for those military hospitals found to be relatively inefficient. The slack is derived from non-binding constraints for inputs: $X_{ko}\theta_o - \sum_i X_{ki}\lambda_i$. The degree of slacks varies depending on input types and hospitals. Overall, the slacks occurred most frequently for FTE (54/99), and its dispersion was the largest among the three inputs (coefficient of variation=2.1). The results indicate that MHS hospitals could reduce 2% to 30% of the current input levels without changing their efficiency performance. If these excessive resources, especially FTEs, can be re-allocated across the military hospital network, the system may be able to address increasing demands with lower costs. Also, the MHS hospitals that can use these additional resources may be able to produce better performance outcomes.

Table 2: Summary of *DEA* Slacks for Input Variables

	Min.	*Average*	*Max*	*StDev*
ENROLL	0	588	10990	1465
FTE	0	20	254	40.9
EXP	0	2.3M	38.6M	6.3M

Table 3 summarizes the amount of each input that inefficient military hospitals can potentially decrease to achieve technical efficiency. The amount of excessive inputs can be derived by $X_{ki}(1-\theta_i)$. Since it was assumed that military hospitals do not have a control over $ENROLL$, it was considered only for the two inputs, FTE and EXP. The summary shows that it would be possible for a military hospital to decrease FTE by 1.6 to 1179.2 and decrease EXP by \$0.14 million to \$157 million, without decreasing any of its outputs.

Table 3: Summary of *DEA* Excessive Inputs

	Min.	*Average*	*Max*	*StDev*
FTE	1.6	226.3	1179.2	252.8
EXP	0.14M	26.3M	157.3M	31.1M

The results indicate that the inefficient MHS hospitals need to reduce 1% to 57% of their current FTEs and expenditures to achieve technical efficiency. Not all of those MHS hospitals may be able to take drastic actions in order to achieve efficiency. However, this analysis provide a guideline for policies and priorities for an effective resource allocation. Automatic re-allocation of inputs may reduce this variability in inputs while increasing the overall health system performance.

3.2 Multi-Objective Auto-Optimization Model (*MAOM*) Results

Even though *DEA* is a valuable tool to understand the hospital efficiencies and conduct manual sensitivity analyses using the slack and reduced costs, *DEA* fails to provide any direct decision support for decision-makers. Therefore, we employ the *MAOM* for automatic re-allocation of system inputs (FTE, EXP) across all 128 hospitals within the MHS to maximize overall performance.

This resource allocation-based optimization model is tested with a minimum efficiency score $r = 0.900$ for all hospitals. The resource changes for inputs FTE and EXP are limited from both sides to $\gamma = 25\%$ to prevent major adjustments. Note that these values of r and γ were chosen based upon feedback received by the decision-makers at the DHA; these values were also used in Bastian et al. [3] and Ekin et al. [19]. Also, side constraints are added due to the non-discretionary nature of the input variable *ENROLL* ($\delta_{k=ENROLL,i} = 0 \quad \forall k \in K, i \in N$). As in the *DEA* analysis, it should be noted that the average value (from 2009-2013) for the input and output values for each hospital were used in the *MAOM* analysis.

As a consequence of the automatic system input resource re-allocations across the MHS, 81 hospitals achieved technically efficiency (Pareto optimally) while 47 remained technically inefficient. Figure 2 shows the distribution of efficiency scores after using the *MAOM* for automatic resource re-allocation of staffing and funding. The histogram shows extreme left-skew, with a mean and median hospital efficiency score of 0.96 and 1.0, respectively.

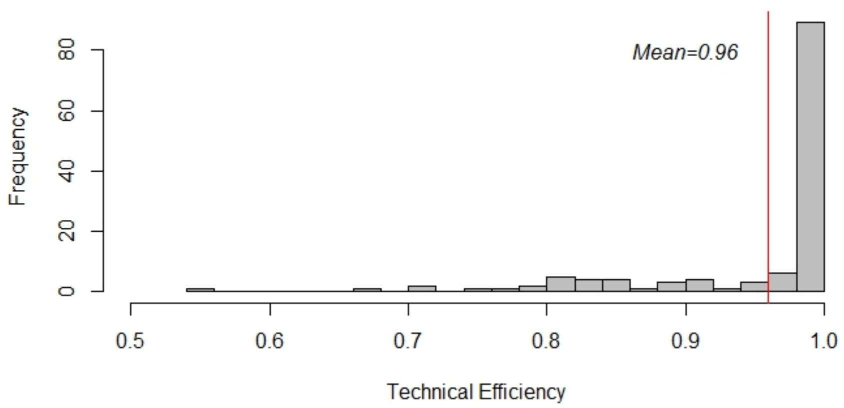


Fig. 2: Distribution of *MAOM* Efficiency Scores

Upon automatically re-allocating FTE and EXP input resources, there is considerable improvement over the *DEA* results in terms of system-wide efficiency, as well as increases in efficiency for each hospital. This improvement can be seen by comparing Figures 1 and 2, with associated efficiency averages of 0.79 and 0.96, respectively. As indicated by this increase of mean efficiency, the resource re-allocation leads to significant performance improvement across the MHS.

Figure 3 shows the change in initial and final input resources for FTE and EXP for each hospital. For points above the red/blue horizontal line, this indicates a positive resource re-allocation (i.e., increase in resources) for that particular input and hospital. Meanwhile, points above the red/blue horizontal line suggest a negative resource re-allocation (i.e., reduction in resources) for that particular input and hospital.

The plot suggests an interesting trend where hospitals who had a relatively large resource re-allocation for FTE also had a relatively large resource re-allocation for EXP (e.g., hospitals H1 to H33). It appears that this shift trend is necessary to optimize efficiency for that particular hospital and the system overall. Note that this is not the case for every hospital, but the results do indicate a general pattern.

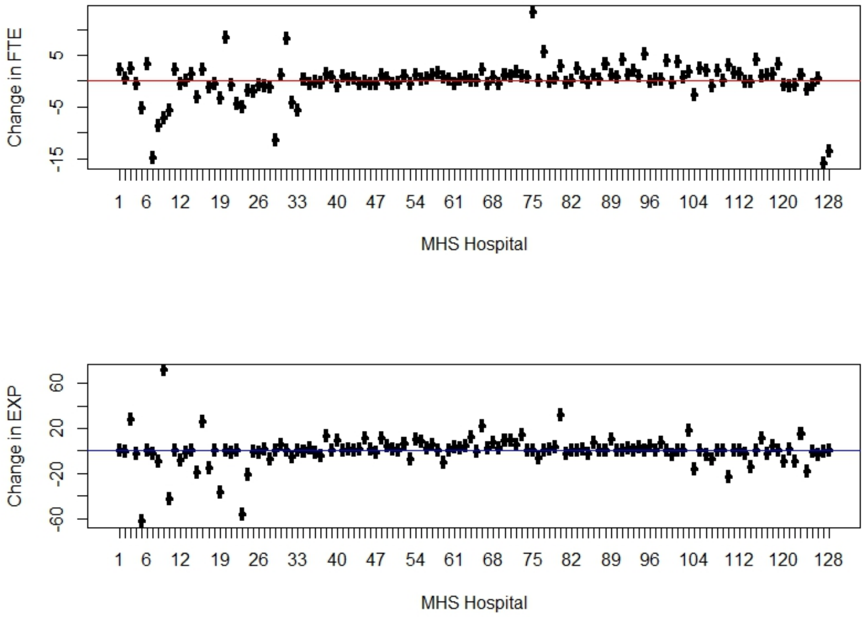


Fig. 3: *MAOM* Resource Re-allocations for MHS Staffing and Funding

3.3 Stochastic Multi-Objective Auto-Optimization Model (*SMAOM*) Results

As mentioned previously, the limitation of the *MAOM* is that it cannot help DHA decision-makers automatically re-allocate input resources across MHS hospitals for different levels of resource uncertainty. The proposed *SMAOM* method, however, addresses this limitation by dropping the assumption of deterministic health system input and output resources, thereby accounting for uncertainty in the data parameters. Therefore, we employ the *SMAOM* for automatic re-allocation of stochastic health system inputs (FTE, EXP) to optimize overall system performance across the 128 military hospitals within the health system.

As in the *MAOM*, this resource allocation-based optimization model is tested with a minimum efficiency score $r_i = r_{min} = 0.900$ for all hospitals $i \in N$ and $r_{max} = 1$. Further, the resource changes for stochastic inputs FTE and EXP are again limited from both sides to $\gamma = 25\%$ to prevent major adjustments, side constraints are added due to the non-discretionary nature of the input variable *ENROLL* ($\delta_{k=ENROLL,i} = 0 \quad \forall k \in \widetilde{K}, i \in N$). Upon statistical inspection of the probability distribution for each of the inputs and outputs associated with each DMU from the 2009 – 2013 data, normal distributions provided adequate fits. Within the stochastic sampling mechanism of the *SMAOM*, each scenario maps directly to a sample realization of the stochastic inputs $\widetilde{x}_{ki}$ and outputs $\widetilde{y}_{ji}$, which are simultaneously, randomly drawn with replacement from respective normal distributions using the empirical means and standard deviations computed from the data.

In running the *SMAOM*, the scenario-based Monte Carlo simulation stochastic programming solution method uses $s = 30$ sample scenarios. As a consequence of the automatic system input resource re-allocations under uncertainty across the MHS, 38 hospitals achieved stochastic Pareto optimally while 90 hospitals remained stochastic inefficient. Figure 4 shows the distribution of expected hospital efficiency scores after using the *SMAOM* for stochastic, automatic resource re-allocation of staffing and funding. The histogram shows extreme left-skew, with a mean expected hospital efficiency score of 0.93 and median expected hospital efficiency score of 0.97.

To better visualize the variability of these expected hospital efficiency scores, Figure 5 displays a plot of these expected efficiencies for each of the 128 military hospitals along with a 95% confidence interval (shown in red).

We used Figure 5 to understand better the impact of uncertainty for each MHS hospital by examining the variability of the technical efficiency scores. In particular, this plot helped us visualize whether or not *MAOM* efficiency scores were between the lower and upper bounds of the 95% confidence interval, which was useful in comparing *SMAOM* with *MAOM*. Figure 6 shows the change in expected initial and final stochastic input resources of FTE and EXP for each hospital.

As before, points above the red/blue horizontal line suggest a positive resource re-allocation for that particular input and hospital, while points above the red/blue horizontal line indicate a negative resource re-allocation. The

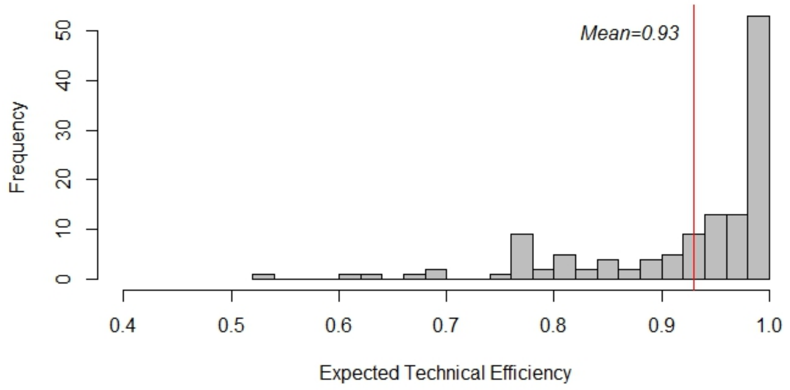


Fig. 4: Distribution of *SMAOM* Expected Efficiency Scores

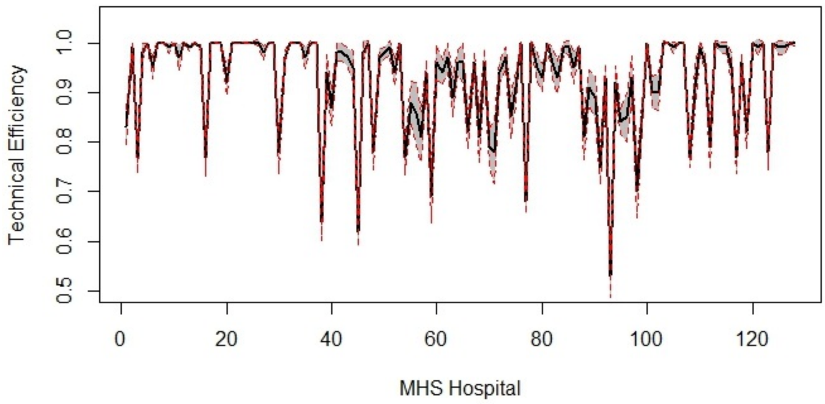


Fig. 5: *SMAOM* Expected Efficiencies with 95% CI

plot suggests a similar trend as in the *MAOM* results, but there does appear to be slightly more overall variability in the total amount of resource re-allocation for both FTE and EXP.

3.4 Comparison of *DEA*, *MAOM* and *SMAOM* Results

In comparing the results of the MHS hospital efficiency scores computed using the proposed methods, the histograms in Figures 1, 2, and 4 suggest that the resource re-allocations performed by the *MAOM* led to significantly more Pareto optimal hospitals compared to *DEA*. In addition, the resource re-allocations suggested by the *SMAOM* also led to an improvement in overall health system performance compared to *DEA*. However, it should be noted that the *MAOM* resulted in a greater number of Pareto optimal hospitals compared to the *SMAOM*.

These results are echoed in Figure 7, which illustrates the difference in efficiency score for each hospital in the MHS. Clearly, the automatic staffing and funding adjustments made by both the *MAOM* and *SMAOM* resource allocation-based optimization methods improved the technical efficiency of each hospital, as well as the overall performance of the system.

We ran Pearson's correlation for the efficiency scores generated by all three models. In all cases, the correlation was positive and moderate to strong, indicating convergent validity of the three methods. The weakest correlation (0.674) was between *DEA* and *MAOM*. Since *SMAOM* is a stochastic variant of *MAOM*, we were not surprised to find the high correlation (0.909) between the two models. In addition, we performed additional runs of the *SMAOM* with the number of sample scenarios $s = 50, 100, 150, 200$. We found that the correlation between the *MAOM* efficiency scores and expected *SMAOM* efficiency scores improved as the value of s increased. This occurred because as the sample size increased, there was a reduction in the standard error of the expected stochastic efficiencies. Table 4 shows the result of the Pearson's correlation analysis.

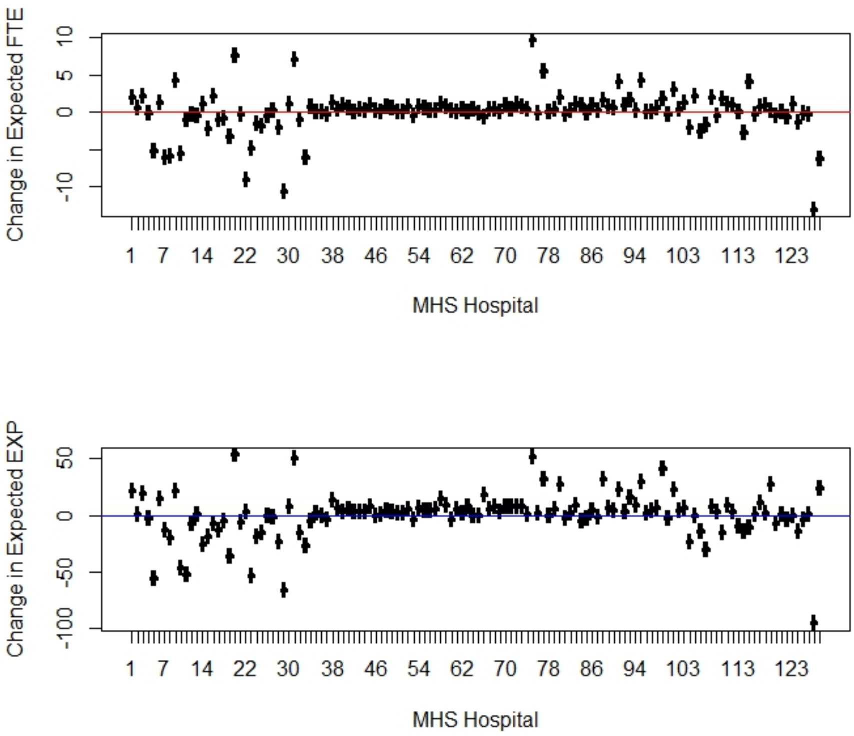


Fig. 6: *SMAOM* Expected Resource Re-allocations for MHS Staffing and Funding

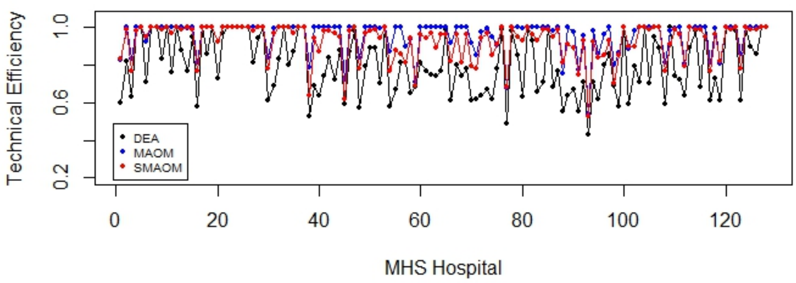


Fig. 7: Comparison of *DEA*, *MAOM* and *SMAOM* Efficiency by Military Hospital

Table 4: Pearson's Correlation Among the Efficiency Scores for the Models

	DEA	*MAOM*	*SMAOM*
DEA	1	.674*	.752*
MAOM		1	.909*
SMAOM			1
$^*p < .001$			

To confirm the convergent validity between the *MAOM* and *SMAOM*, we also wanted to understand better the differences between the *MAOM* and *SMAOM* efficiency scores. Figure 8 provides a histogram of the absolute differences between *MAOM* and *SMAOM* efficiency scores. This extremely right-skewed density confirms the convergent validity as illustrated by the relatively minor differences between the efficiencies. Figure 9 shows two scatterplots of the differences between *MAOM* and *SMAOM* efficiency scores against measures of hospital size. Regarding hospital size, the left sub-figure shows the average hospital enrollment population, while the right sub-figure shows the average hospital outpatient workload. Both sub-figures show a similar pattern regarding the relationship between the *MAOM*/*SMAOM* technical efficiency score differences and hospital size, and both sub-figures have a similar right-skew to that shown in Figure 8. These results suggest that smaller-sized hospitals have slightly greater absolute differences of *MAOM* and *SMAOM* efficiency scores compared to larger-sized hospitals.

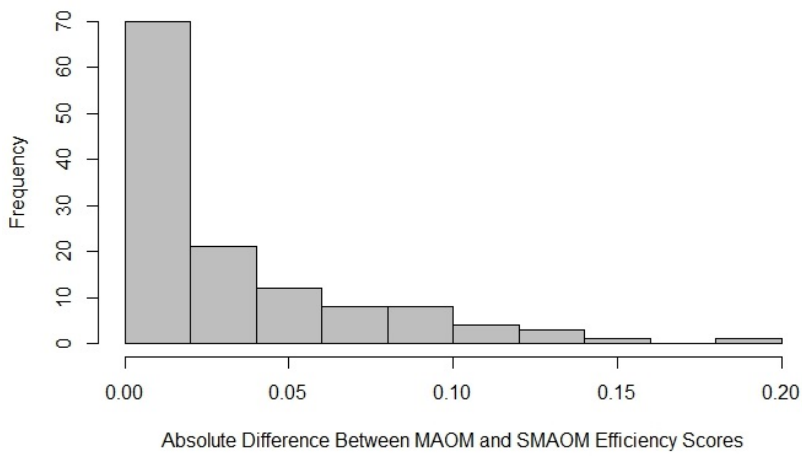


Fig. 8: Histogram of Differences between *MAOM* and *SMAOM* Efficiency Scores

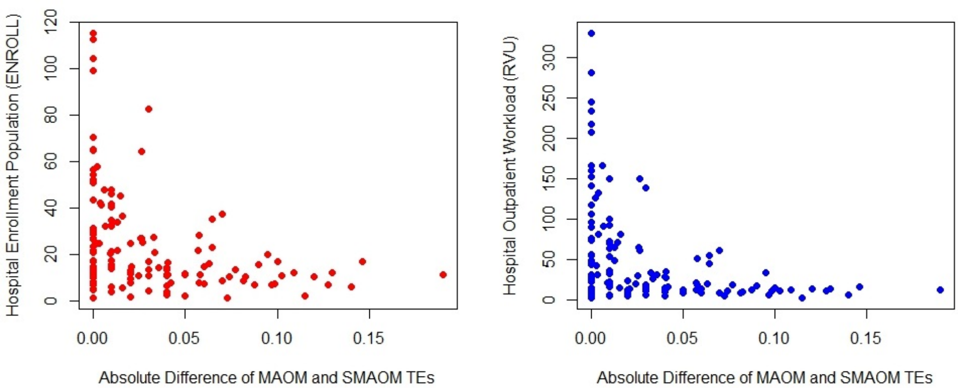


Fig. 9: Scatterplots of Differences between *MAOM* and *SMAOM* Efficiencies Against Hospital Size

We also evaluated the directional congruence of the recommended changes of *MAOM* versus those produced by *SMAOM* for both expenditures and full-time equivalents. The congruence for expenditures was 80.47%, indicating that nearly 81% of all recommendations were identical in direction. Using a two-tailed sign test with a null assumption of a 50% match, we found that the Binomial probability of having 103 successes in 128 trials under the null was less than 0.001. The directional match of staffing changes was 79.7%, and the two-tailed binomial probability was less than 0.001. So the models produce very similar results when looking at both *EXP* and *FTE* changes in terms of directionality.

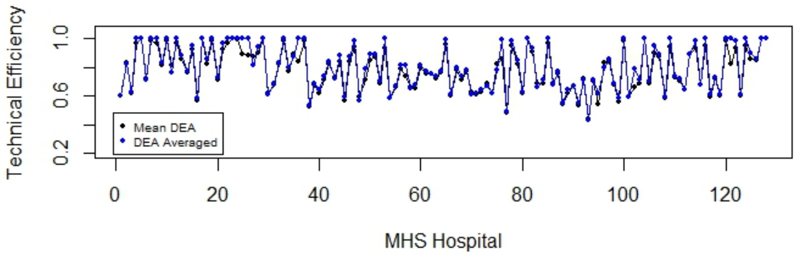
Given that we performed cross-sectional analysis on averaged values of inputs and outputs for the *DEA* and *MAOM* analyses, we also wanted to compare these results to the efficiency scores generated for each year (2009 – 2013) and each of the 128 military hospitals. This year-by-year comparison with the averaged results, along with the expected efficiency and 95% confidence interval lower (LB) and upper bounds (UB) computed from the *SMAOM* analysis, is shown in Table 5. We found that nearly 60% of the *MAOM* efficiency scores fell strictly within the lower and upper bounds of the 95% confidence interval computed from the *SMAOM* results. Note, however, that those efficiency scores falling outside of this confidence interval did so trivially.

For the *DEA* results in Table 5, there is no statistically significant difference between the mean efficiency score (over the five years) and the averaged efficiency score for each of the military hospitals (p-value = 0.28). For the *MAOM* results in Table 5, we again found no statistically significant difference between the mean efficiency score

Table 5: Year-by-year Comparison of *DEA*, *MAOM* and *SMAOM* Analyses

	DEA							MAOM							SMAOM		
DMU	**2009**	**2010**	**2011**	**2012**	**2013**	**Mean**	**Averaged**	**2009**	**2010**	**2011**	**2012**	**2013**	**Mean**	**Averaged**	**Mean**	**LB**	**UB**
H1	0.57	0.66	0.52	0.58	0.69	0.60	0.60	0.66	0.91	0.70	0.83	1.00	0.82	0.83	0.83	0.79	0.87
H2	0.73	0.76	0.73	0.92	1.00	0.83	0.82	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	1.00
H3	0.65	0.66	0.58	0.59	0.64	0.62	0.63	0.75	0.91	0.73	0.85	0.88	0.82	0.83	0.77	0.74	0.80
H4	1.00	0.85	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.96	1.00
H5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H6	0.80	0.61	0.66	0.71	0.81	0.72	0.71	1.00	0.79	0.86	1.00	0.99	0.93	0.92	0.95	0.93	0.97
H7	0.97	0.95	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H8	0.87	1.00	0.97	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H9	0.87	0.88	0.75	0.84	0.73	0.81	0.83	1.00	1.00	0.96	1.00	0.95	0.98	1.00	0.99	0.98	1.00
H10	0.96	1.00	0.93	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H11	1.00	0.89	0.86	0.64	0.68	0.81	0.76	1.00	1.00	1.00	0.93	1.00	0.99	1.00	0.97	0.94	1.00
H12	0.86	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H13	0.83	0.97	0.81	0.86	0.85	0.86	0.88	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	1.00
H14	0.76	0.87	0.76	0.71	0.69	0.76	0.77	1.00	0.99	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00
H15	0.87	0.90	0.90	1.00	0.94	0.92	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	1.00
H16	0.59	0.64	0.55	0.57	0.52	0.57	0.58	0.73	0.91	0.78	0.76	0.64	0.76	0.81	0.77	0.73	0.81
H17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H18	0.78	0.68	0.79	0.96	0.90	0.82	0.86	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H19	0.89	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H20	0.70	0.73	0.67	0.71	0.74	0.71	0.73	0.87	0.86	0.82	0.88	0.98	0.88	0.92	0.92	0.89	0.95
H21	0.79	0.94	0.92	0.99	0.98	0.92	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H22	0.96	0.91	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H24	0.97	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H25	1.00	1.00	0.83	0.78	0.85	0.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H26	0.84	0.93	0.84	0.89	0.91	0.88	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.01
H27	0.77	0.82	0.75	1.00	1.00	0.87	0.81	0.92	1.00	0.99	1.00	0.99	0.98	1.00	0.98	0.97	0.99
H28	0.82	0.95	1.00	0.84	0.89	0.90	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H30	0.68	0.60	0.57	0.67	0.57	0.62	0.61	0.73	0.77	0.84	1.00	0.73	0.81	0.84	0.78	0.74	0.82
H31	0.54	0.59	0.67	0.78	0.82	0.68	0.69	0.73	0.80	0.97	1.00	1.00	0.90	1.00	0.97	0.96	0.98
H32	0.79	0.76	0.84	0.91	0.82	0.82	0.83	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H33	1.00	0.98	1.00	1.00	0.93	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H34	0.75	0.82	0.82	0.70	0.74	0.77	0.80	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H35	0.73	0.93	0.87	0.92	0.98	0.89	0.87	0.85	1.00	1.00	1.00	1.00	0.97	1.00	0.97	0.95	0.99
H36	0.87	0.76	0.81	0.83	0.94	0.84	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H37	0.94	1.00	0.97	0.99	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H38	0.55	0.54	0.52	0.50	0.48	0.52	0.53	0.73	0.77	0.84	0.72	0.65	0.74	0.79	0.64	0.60	0.68
H39	0.72	0.67	0.68	0.70	0.63	0.68	0.69	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.94	0.90	0.98
H40	0.54	0.61	0.63	0.64	0.70	0.62	0.64	0.76	0.95	1.00	1.00	1.00	0.94	1.00	0.87	0.83	0.91
H41	0.62	0.72	0.75	0.76	0.71	0.71	0.74	0.96	1.00	1.00	1.00	1.00	0.99	1.00	0.98	0.96	1.00
H42	0.79	0.78	0.81	0.86	0.84	0.82	0.84	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.96	1.00
H43	0.65	0.70	0.75	0.83	0.70	0.73	0.72	0.93	1.00	1.00	1.00	1.00	0.99	1.00	0.97	0.95	0.99
H44	0.87	0.81	0.87	0.86	0.80	0.84	0.88	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.95	0.92	0.98
H45	0.56	0.53	0.53	0.63	0.62	0.57	0.59	0.68	0.67	0.50	0.77	0.75	0.67	0.72	0.62	0.59	0.65
H46	0.83	0.84	0.87	0.85	0.82	0.84	0.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.95	1.01
H47	0.85	0.96	0.99	1.00	0.88	0.94	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H48	0.58	0.59	0.56	0.68	0.53	0.59	0.57	0.68	0.71	0.82	0.91	0.75	0.77	0.84	0.78	0.74	0.82
H49	0.69	0.71	0.61	0.69	0.87	0.71	0.79	1.00	1.00	0.99	1.00	1.00	1.00	1.00	0.97	0.95	0.99
H50	0.85	0.81	0.92	0.83	0.85	0.85	0.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.97	0.99
H51	0.86	0.88	0.91	0.90	0.79	0.87	0.89	1.00	1.00	1.00	1.00	0.97	0.99	1.00	0.99	0.98	1.00
H52	0.60	0.68	0.71	0.74	0.73	0.69	0.70	0.81	1.00	1.00	1.00	1.00	0.96	1.00	0.94	0.91	0.97
H53	0.94	0.84	0.86	1.00	1.00	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H54	0.58	0.63	0.56	0.61	0.53	0.58	0.58	0.72	0.96	0.87	0.90	0.60	0.81	0.87	0.77	0.73	0.81
H55	0.58	0.69	0.66	0.70	0.65	0.66	0.67	0.82	1.00	1.00	1.00	0.93	0.95	1.00	0.88	0.84	0.92
H56	0.81	0.80	0.86	0.78	0.72	0.79	0.81	1.00	1.00	1.00	0.94	0.73	0.93	1.00	0.86	0.80	0.92
H57	1.00	1.00	0.59	0.55	0.56	0.74	0.81	0.98	0.95	0.93	0.73	0.65	0.85	0.90	0.81	0.76	0.86
H58	0.61	0.63	0.66	0.73	0.65	0.66	0.65	0.96	0.95	1.00	1.00	0.99	0.98	0.94	0.94	0.91	0.97
H59	0.71	0.66	0.60	0.65	0.65	0.65	0.68	0.91	0.79	0.64	0.66	0.61	0.72	0.71	0.69	0.64	0.74
H60	0.82	0.82	0.80	0.81	0.75	0.80	0.81	0.87	0.94	1.00	1.00	1.00	0.96	1.00	0.96	0.92	1.00
H61	0.73	0.82	0.77	0.75	0.68	0.75	0.77	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.94	0.91	0.97
H62	0.65	0.81	0.74	0.80	0.74	0.75	0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	0.94	1.00
H63	0.67	0.69	0.72	0.78	0.76	0.72	0.74	0.91	1.00	1.00	1.00	1.00	0.98	1.00	0.89	0.85	0.93
H64	0.54	0.70	0.73	0.85	1.00	0.76	0.77	0.72	1.00	1.00	1.00	1.00	0.94	1.00	0.96	0.93	0.99
H65	0.95	1.00	0.92	0.93	1.00	0.96	0.99	0.75	1.00	1.00	1.00	1.00	0.95	1.00	0.96	0.92	1.00
H66	0.63	0.63	0.58	0.61	0.57	0.60	0.61	0.82	0.92	0.86	0.71	0.79	0.82	0.92	0.82	0.79	0.85
H67	0.78	0.72	0.82	0.83	0.74	0.78	0.80	1.00	0.99	1.00	1.00	1.00	1.00	1.00	0.96	0.94	0.98
H68	0.69	0.68	0.73	0.75	0.71	0.71	0.74	1.00	0.97	1.00	1.00	0.87	0.97	1.00	0.81	0.76	0.86
H69	0.76	0.70	0.80	0.81	0.77	0.77	0.78	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96	0.93	0.99
H70	0.59	0.67	0.60	0.72	0.56	0.63	0.61	0.79	0.88	0.83	0.98	0.82	0.86	0.92	0.79	0.76	0.82
H71	0.62	0.68	0.64	0.59	0.54	0.61	0.62	0.74	0.86	0.98	0.84	0.68	0.82	0.85	0.78	0.72	0.84
H72	0.62	0.63	0.61	0.67	0.62	0.63	0.64	0.96	1.00	1.00	1.00	0.99	0.99	0.97	0.94	0.92	0.96
H73	0.62	0.66	0.69	0.73	0.73	0.69	0.67	0.96	1.00	1.00	0.99	1.00	0.99	1.00	0.97	0.95	0.99
H74	0.66	0.61	0.63	0.57	0.61	0.62	0.62	0.98	0.76	0.94	0.92	0.91	0.90	0.95	0.85	0.81	0.89
H75	1.00	0.82	0.74	0.75	0.77	0.82	0.78	1.00	1.00	0.86	0.76	0.78	0.88	0.90	0.91	0.88	0.94
H76	0.83	0.84	0.92	0.86	0.83	0.86	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H77	0.58	0.48	0.43	0.45	0.47	0.48	0.49	0.84	0.67	0.57	0.62	0.60	0.66	0.68	0.68	0.65	0.71
H78	0.99	1.00	1.00	0.98	0.78	0.95	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01
H79	0.82	0.81	0.82	0.86	0.78	0.82	0.85	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.95	0.92	0.98
H80	0.67	0.64	0.58	0.60	0.61	0.62	0.63	0.99	0.98	0.79	0.82	0.97	0.91	0.99	0.93	0.90	0.96
H81	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H82	0.92	0.96	0.86	0.88	0.91	0.91	0.93	0.89	1.00	1.00	1.00	1.00	0.98	1.00	0.96	0.93	0.99
H83	0.98	0.70	0.56	0.59	0.63	0.69	0.66	1.00	1.00	0.83	0.86	0.99	0.94	1.00	0.93	0.90	0.96
H84	0.59	0.64	0.75	0.73	0.75	0.69	0.71	0.95	1.00	1.00	1.00	1.00	0.99	1.00	0.99	0.98	1.00
H85	1.00	1.00	0.92	1.00	0.91	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	1.00
H86	0.64	0.69	0.69	0.71	0.70	0.69	0.68	0.98	0.99	1.00	0.99	0.96	0.98	0.98	0.95	0.92	0.98
H87	0.61	0.78	0.77	0.80	0.81	0.75	0.77	0.98	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99
H88	0.49	0.57	0.54	0.58	0.54	0.54	0.55	0.63	0.69	0.69	0.81	0.79	0.72	0.75	0.81	0.76	0.86
H89	0.60	0.61	0.64	0.67	0.58	0.62	0.64	0.86	1.00	1.00	1.00	0.92	0.96	1.00	0.91	0.87	0.95
H90	0.62	0.65	0.70	0.73	0.65	0.67	0.67	0.77	0.93	1.00	1.00	1.00	0.94	0.97	0.89	0.85	0.93
H91	0.52	0.49	0.51	0.55	0.57	0.53	0.55	0.64	0.67	0.73	0.81	0.85	0.74	0.78	0.75	0.71	0.79
H92	0.75	0.73	0.67	0.71	0.72	0.72	0.71	0.97	0.97	1.00	0.89	0.98	0.96	0.95	0.93	0.90	0.96
H93	0.54	0.42	0.42	0.43	0.37	0.44	0.43	0.75	0.59	0.64	0.61	0.48	0.61	0.54	0.53	0.49	0.57
H94	0.68	0.67	0.67	0.74	0.76	0.70	0.71	0.93	1.00	1.00	1.00	0.98	0.98	0.98	0.92	0.89	0.95
H95	0.55	0.55	0.49	0.54	0.55	0.54	0.62	0.84	0.78	0.77	0.85	0.89	0.83	0.86	0.84	0.81	0.87
H96	0.90	0.92	0.80	0.77	0.74	0.83	0.80	1.00	0.86	0.80	0.99	0.72	0.87	0.96	0.85	0.80	0.90
H97	0.82	0.92	0.87	0.81	0.76	0.84	0.86	1.00	1.00	1.00	1.00	0.84	0.97	1.00	0.93	0.89	0.97
H98	0.58	0.69	0.70	0.70	0.72	0.68	0.69	0.58	0.79	0.79	0.93	0.76	0.77	0.80	0.70	0.65	0.75
H99	0.57	0.56	0.57	0.58	0.53	0.56	0.58	0.80	0.79	0.89	0.84	0.85	0.83	0.86	0.85	0.82	0.88
H100	1.00	1.00	1.00	1.00	0.90	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H101	0.62	0.54	0.53	0.59	0.65	0.59	0.59	0.93	0.82	0.78	0.89	1.00	0.88	0.89	0.90	0.87	0.93
H102	0.72	0.74	0.65	0.65	0.56	0.66	0.79	1.00	1.00	1.00	1.00	0.87	0.97	0.98	0.90	0.86	0.94
H103	0.73	0.78	0.66	0.64	0.62	0.69	0.71	1.00	1.00	0.95	0.99	0.96	0.98	1.00	1.00	1.00	1.00
H104	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H105	0.78	0.70	0.69	0.64	0.62	0.69	0.70	1.00	1.00	0.99	1.00	1.00	1.00	1.00	0.99	0.98	1.00
H106	0.89	0.96	0.80	0.91	0.95	0.90	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H107	0.93	0.85	0.85	0.84	0.90	0.87	0.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H108	0.64	0.59	0.54	0.58	0.53	0.58	0.59	0.75	0.82	0.79	0.81	0.70	0.77	0.81	0.77	0.75	0.79
H109	1.00	1.00	1.00	1.00	0.70	0.94	1.00	0.85	1.00	1.00	0.93	0.51	0.86	0.98	0.91	0.86	0.96
H110	0.73	0.76	0.74	0.73	0.70	0.73	0.74	1.00	1.00	1.00	0.97	0.97	0.99	0.99	0.99	0.98	1.00
H111	0.71	0.74	0.67	0.70	0.69	0.70	0.72	0.97	1.00	0.87	0.94	0.97	0.95	1.00	0.96	0.94	0.98
H112	0.63	0.62	0.62	0.69	0.64	0.64	0.64	0.75	0.69	0.81	0.82	0.67	0.75	0.81	0.79	0.74	0.84
H113	0.93	0.94	0.84	0.90	0.84	0.89	0.89	0.99	0.98	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00
H114	1.00	0.98	0.86	0.92	0.89	0.93	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	1.00
H115	0.66	0.67	0.62	0.67	0.79	0.68	0.68	1.00	0.99	1.00	1.00	1.00	1.00	1.00	0.99	0.98	1.00
H116	1.00	0.85	0.88	1.00	1.00	0.95	1.00	0.95	0.94	0.98	1.00	0.97	0.97	1.00	0.95	0.92	0.98
H117	0.58	0.59	0.59	0.60	0.61	0.59	0.61	0.63	0.71	0.74	0.79	0.85	0.74	0.81	0.77	0.74	0.80
H118	0.76	0.69	0.71	0.73	0.73	0.72	0.73	0.93	1.00	1.00	1.00	0.99	0.98	1.00	0.96	0.94	0.98
H119	0.67	0.62	0.57	0.60	0.54	0.60	0.61	0.92	0.77	0.62	0.78	0.79	0.78	0.81	0.82	0.79	0.85
H120	1.00	1.00	0.98	0.92	0.98	0.98	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H121	0.70	0.76	0.96	0.90	0.77	0.82	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	1.01
H122	0.89	0.94	0.87	0.95	0.98	0.93	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H123	0.64	0.66	0.55	0.57	0.56	0.60	0.61	0.84	0.83	0.82	0.83	0.80	0.82	0.86	0.78	0.74	0.82
H124	1.00	1.00	1.00	0.92	0.84	0.95	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00
H125	0.90	0.98	0.92	0.76	0.76	0.86	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	1.00
H126	1.00	0.85	0.82	0.85	0.75	0.85	0.86	0.97	1.00	1.00	1.00	0.98	0.99	1.00	0.99	0.98	1.00
H127	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H128	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01

(over the five years) and the averaged efficiency score for each of the military hospitals (p-value = 0.12). Note that visual inspection of Figure 10 confirms the trivial differences between mean of the year-by-year efficiency scores and the averaged efficiency scores for both the *DEA* and *MAOM* analyses.



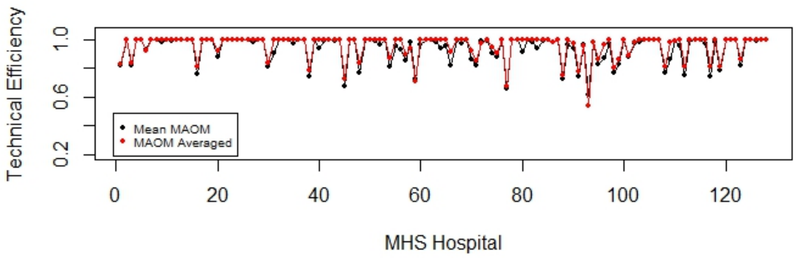


Fig. 10: Comparison of Year-by-year and Averaged *DEA* and *MAOM* Efficiency Scores

4 Concluding Remarks

This paper introduced the Stochastic Multi-Objective Auto-Optimization Model for resource allocation decision-making in fixed-input health systems under uncertainty. The efficient allocation of resources for health systems is paramount due to the expectation to deliver top-notch health care in the face of rising costs. The proposed model accommodates uncertainty in health system parameters and automatically allocates resources in order to maximize the system-wide performance of a fixed-input health system.

The *SMAOM*, *MAOM* and classic *DEA* models were compared to illustrate how the automatic allocation of system input resources can readily improve technical efficiency scores of hospitals in a fixed-input health system. Annual hospital level data from 2009 – 2013 for 128 MHS hospitals were used to run a computational experiment. Results of the experiment highlight an expected system-wide increase of 18% technical efficiency via the *SMAOM*. The results also indicated that nearly all of the *MAOM* efficiency scores fell within (or extremely near) the 95% confidence interval produced by the *SMAOM*, suggesting that adding stochasticity to the *MAOM* makes the results less sensitive to outlier DMUs, which straight *DEA* and *MAOM* methods are subject to.

The primary motivation of this research was to support the strategic-level objective of the DHA to drive greater integration of clinical and business processes across the MHS by implementing shared services with common measurement outcomes and exercising management responsibility for joint shared services [15]. The *SMAOM* proffered in this research was designed specifically to support DHA managers in optimizing system-wide performance by sharing input resources amongst the three medical services (Army, Navy, Air Force) in an integrative fashion. As a result, the *SMAOM* directly supports DHA management and policy-makers in helping determine effective and efficient options for sharing MHS input resources such that all three medical services are held responsible for shared health service support.

The *SMAOM* helps achieve full operating capability for integrated MHS health services to improve MHS-wide standardization, efficiency, and jointness. It plays a critical role in facilitating business planning and performance management for the enhanced Multi-Service Markets (eMSMs) around the country. Within each eMSM, an appointed market manager has the authority to: 1) manage the allocation of the budget for the market; 2) direct common clinical and business functions for the market; 3) optimize readiness to deploy medically-ready forces; 4) direct the workload and workforce among market medical treatment facilities; and 5) develop, execute and monitor the business performance plan [15]. Given these responsibilities of the market manager, the *SMAOM* is extremely valuable for direct support of performance planning and execution in that it provides recommendations for the efficient and effective allocation of funding and staffing in a way that optimizes the overall performance.

The *SMAOM* developed in this research has significant managerial implications for senior DHA leaders and MHS decision-makers in that it is useful as decision support to: 1) promote more effective and efficient healthcare operations through enhanced enterprise-wide shared services; 2) match personnel, infrastructure, and funding to current missions, future missions, and population demand; and 3) create enhanced value in military medical markets using an integrated approach in short-term business plans. Hence, the results of this work help develop, shape and empower strategically informed leaders, which is imperative to the success of the MHS.

One limitation of the proposed model is the non-linearity of the *SMAOM* model. We acknowledge that the *SMAOM* would not be the best approach for a ”real-time” decision support tool. However, the planning horizon for large fixed-input health systems, i.e. yearly budgets and planned allocations, indicates the maximum solution time of approximately 30 minutes is acceptable for this problem. Another limitation of this model is that decision-makers responsible for the fixed-input health system must be technically proficient to set up and solve the model.

Future work will explore more dynamic models to accommodate scenarios in which health system inputs vary over time. For example, a small mobile field hospital may be re-located frequently to natural disaster sites and demand different system resources for each disaster. Future work will also identify other non-health system applications to apply the *SMAOM*. Public resources, such as fire stations, police stations and schools, are also in need of the efficient allocation of resources and have uncertain system parameters.

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